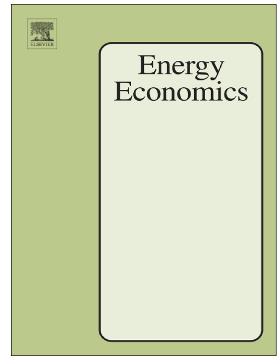

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Integrating Real Options Analysis with long-term electricity market models

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Highlights

- In liberalized electricity markets, ongoing uncertainties play a major role to delay the decision-making of new power generation investments.
- A valuation framework of power plant investments, based on Real Options Analysis, is integrated with a long-term electricity market model.
- The decision-making model considers that the addition of new power capacity is guided by the economic value of the option to defer projects under uncertainty.
- The main contribution is the integration of decision and option valuation theory in a long-term model for reproducing investment cycles observed in electricity markets.

Abstract

In liberalized electricity markets, the investment postponement option is deemed decisive for understanding the addition of new generating capacity. Basically, it refers to the possibility for investors to postpone projects for a period while waiting for the arrival of new and better information about the market evolution. When such development involves major uncertainties, the generation business becomes riskier, and the investors' *"wait-and-see"* behavior might limit the timely addition of new power plants. In that sense, the literature provides solid empirical evidence about the occurrence of construction cycles in the deregulated electricity industry. However, the strategic flexibility inherent to the option to defer new power plants has not been yet rigorously incorporated to investment signals in existing market models. Therefore, this paper proposes a novel methodology to assess the long-term development of liberalized power markets based on a more realistic approach for valuing generation investments. The work is based on a stochastic dynamic market model, built upon System Dynamics simulation approach. The decision-making framework considers that the addition of new capacity is driven by the economic value of the strategic flexibility associated to defer investments under uncertainties. Thus, the value of the postponement option is quantified in monetary terms through Real Options Analysis. Simulations confirm the cyclical behavior of the energy-only market in the long run, as suggested by the empirical evidence found in the literature. In addition, sensitivity analysis regarding some relevant exogenous variables depicts an even more fluctuating evolution of the capacity due to the combination of strong demand growth rates with large volatilities. Finally, the model validity is assessed through a formal procedure according to the scope of System Dynamics modeling approach.

Key words: Power Generation; Power Market; Real Options; Stochastic Simulation; Strategic Flexibility; System Dynamics.

List of Abbreviations

ASC	Aggregate Supply Curve
CC	Gas-fired Combined Cycles
CLD	Causal Loop Diagram
CVaR	Conditional Value at Risk
DPE	Dynamic Programming based on the Expected present value.
GT	Gas Turbines
GW	Gigawatt.
HC	Coal-fired power plants
IEA	International Energy Agency.
IRR	Internal Revenue Rate
LDC	Load Duration Curve.
LOLP	Lost of Load Probability
MW	Megawatt.
MWh	Megawatt-hour.
NPV	Net Present Value
PDC	Price Duration Curve.
PI	Profitability Index
ROA	Real Options Analysis
SD	System Dynamics
SFS	Stock-and-Flow Structure
VaR	Value at Risk
VOLL	Value of Lost Load

List of Symbols

Roman symbols

i Subscript to individualize generating technologies [adim]

j Subscript to individualize vintages from each generating technology [adim]

t Dynamic time [month]

d Annual probability of duration of a given load level L [adim]

D_i Annual probability for one MW of new capacity from technology i to operate with market price over own marginal costs [adim]

D_{ij} Annual probability for the capacity of vintage j from technology i to operate with market price over own marginal cost [adim]

D_{def} Annual probability of deficit duration [adim]

dt_i Time increment between the t and M for technology i [month]

€ Symbol of currency (Euro)

FP_i Fuel price of capacity from technology i [€/MWh]

FP_i^r Stochastic realization for the fuel price of capacity from technology i [€/MWh]

g_{FP_i} Growth rate for the fuel price of the capacity from technology i [%/year]

g_m Growth rate for the minimum demand [%/year]

g_M Growth rate for the maximum demand [%/year]

$g_{FP_i}^r$ Stochastic growth rate for the fuel price of capacity from technology i [%/year]

g_L Long-term growth rate for the maximum and minimum demand [%/year]

g_L^r Stochastic growth rate for the maximum and minimum demand [%/year]

g_K^r Stochastic growth rate for the total operating capacity [%/year]

IC_i Investment cost for technology i [€/MW]

K_T Total operating capacity [MW]

K_T^{aval} Available operating capacity [MW]

K_T^r Stochastic realization of expected total operating capacity [MW]

K_T^{UC} Total capacity under construction [MW]

$K_T^{UC^*}$ Total capacity under construction in the long-run equilibrium [MW].

K_i Capacity from technology i [MW]

K_i^{UC} Capacity under construction from technology i [MW]

K_{ij} Capacity from technology i residing in vintage j [MW]

K_{ij}^{cum} Aggregate system capacity up to vintage j from technology i, according to the *dispatch merit order* [MW]

$\dot{K}_{ij}^{in}$ Rate at which capacity enters vintage j from technology i [MW/month]

$\dot{K}_{ij}^{out}$ Rate at which capacity abandons vintage j from technology i [MW/month]

im_i Investment multiplier for technology i [adim]

im_i^{max} Maximum investment multiplier for technology i [adim]

IC_i Investment cost for technology i [€/MW]

$\dot{I}_i$ Investment rate for technology i [MW/month]

$\dot{I}_i^{ref}$ Reference investment rate for technology i [MW/month]

L Load level exceeding an annual duration d [MW]

L_{min} Minimum demand [MW]

L_{min}^{TEST} Test minimum demand [MW]

L_{max} Maximum demand [MW]

L_{max}^{TEST} Test maximum demand [MW]

$\dot{L}_i$ Expected change in the portion of peak load covered by technology i [MW/month]

L^r Stochastic realization of load level exceeding an annual duration d [MW]

M Dynamic *Option Maturity* [month]

$\overline{MC}_{ij}$ Marginal cost of the capacity from technology i residing in vintage j [€/MWh]

$OP_{\bar{T}_i^A}$ Expected stream of operating profits for technology i over $\bar{T}_i^A$ [€/MW]

PI_i Profitability index for technology i [adim]

q Expected availability of generating units [adim]

r Superscript to denoted realizations of stochastic variables [adim]

T_i Lifetime for technology i [year]

$\bar{T}_i^C$ Construction lead-time for technology i [month]

$\bar{T}_i^A$ Amortization period for technology i [month]

V_i^* Optimal investment policy for technology i [€/MW]

V_i^{cont} Continuation value for technology i [€/MW]

V_i^{ex} Exercise value for technology i [€/MW]

Greek symbols

α_i Factor to control the slope of the multiplier curve for technology i [adim]

β_i Factor to define the x-axis position of the multiplier curve for technology i [adim]

$\bar{\eta}_{ij}$ Average efficiency of the capacity from technology i residing in vintage j [adim]

η_{ij}^{in} Efficiency of capacity entering vintage j from technology i [adim]

η_{ij}^{out} Efficiency of capacity abandoning vintage j from technology i [adim]

π_i Annual unitary rents expected by technology i [€/MW·year]

ρ Required revenue rate for each technology [%/year]

ρ^{TEST} Test required revenue rate for each technology [%/year].

φ Risk-free discount rate for each technology [%/year]

Symbols of the *mean-reverting stochastic process*

dg Expected change in a growth rate

dt Time increment

g Growth rate

$\bar{g}$ Long-term (mean) growth rate

η Speed of reversion towards the mean growth rate

σ Volatility of the growth rate

dz Variable following a Wiener Process

ε Normally distributed random variable

θ Correlated random variable

1 Introduction

In the last decades, the evolution towards liberalization of electricity markets has pursued the main objective of improving the economic efficiency of the supply side (IEA, 2003). The deregulation has been founded on strictly market mechanisms, which has led to the unbundling of the industry and the introduction of competition, mainly in the generation segment. Despite many positive outcomes, the cumulated experience after the first stage of reforms has also raised concerns regarding the market attributes that needed to ensure the capacity adequacy (*e.g.* Rudnick *et al.*, 2005; Arango *et al.*, 2006; Joskow, 2006). At first, this seems counterintuitive, since *the theory of spot pricing*, upon which the deregulation is based, theoretically provides sufficient investment incentives in the long-run (Caramanis, 1982). However, it has been reported repeatedly since the beginning of the 1990s that the liberalized power industry is instead prone to suffer construction cycles[1] (Bunn and Larsen, 1992; Bunn and Larsen, 1994).

Many efforts have been put in order to understand the origins of this situation. One of the most accepted explanations poses that the models that have supported deregulation rely on assumptions absent in real markets, such as perfect competition, risk neutrality and full rational behavior of market participants. Indeed, actual markets are likely to deviate from ideal conditions, exhibiting imperfections such as information asymmetry, risk-aversion, herding behavior and bounded rational expectations. Moreover, investors in power plants have the possibility to behave strategically in order to collect extraordinary profits, being prone to exercise market power or to be unresponsive to straight market signals. In that sense, integrating the logic behind the strategic decision-making of new generating capacity has become vital when assessing the long-term market development.

Arango and Larsen (2011) have proposed a comprehensive literature compilation that suggests the appearance of cycles in the construction of investor-owned power plants. Such work presents empirical evidence gathered from over 20 years of reforms in power markets, with the exemplary cases of England and Wales, and Chile. The article explains that the unstable market behavior leads to periods with low reserve margins,

[1] This term refers to the fluctuating development that the capacity is perceived to have exhibited after being deregulated, due to the sequential episodes of over and under-investment

mainly affecting the demand side in terms of high prices and recurrent shortages. However, in times of excess of capacity, generation firms are likely to endure substantial economic losses, and potential bankruptcy. Therefore, the cyclical investment pattern is deemed to pose major concerns for policymakers when assessing the long-run development of the market, since it ultimately has a negative impact on the security of supply (Roques, 2008).

Two factors can be isolated in order to gain insights about the occurrence of construction cycles in the deregulated electricity industry. First, the decision to expand the system has been decentralized to depend on multiple self-oriented, autonomous companies, who attempt to maximize their individual financial profits while managing risks. This defines a market behavior that is dynamic in nature, since it is determined by the actions of individual participants (de Vries and Heijnen, 2008). The second and most important factor indicates that the generation activity has become exposed to several risks, unforeseen in the former regulated industry. Such risks result from the internalization of numerous uncertainties that drive the development of the actual industry in the long run (IEA, 2003; Arango and Larsen, 2011).

Intrinsic characteristics of generation investments, such as capital-intensive, one-stage outlays, long amortization periods and irreversibility, magnify the effects of these factors (Olsina *et al.*, 2006). Given the characteristics of the competitive generation business, investors tend to be *risk-averse* when making decisions (Vázquez *et al.*, 2002). Generally, this rationale suggests that new generating units would be ordered only when large revenues are expected, and conversely decisions would be delayed if the estimated future rents are insufficient or uncertain. Hence, opportunities for investing in the generation sector are no longer of the *now-or-never* type since there is the possibility of waiting for future market conditions to be, at least partially, clarified. This opportunity incorporates one major attribute to the deregulated generation investments, termed the *postponement option* (Olsina *et al.*, 2006). This term explains the investors' willingness to consider the flexibility of deferring new generation projects when facing uncertainty driving the evolution of key market variables (Blanco and Olsina, 2011). It is worth to mention that there is also risk inherent to exercising the postponement option: if rival firms decide to invest before, they would take advantage of the upcoming favorable market conditions.

Despite the abundance of empirical evidence, the literature still lacks a mathematical framework for describing, in theoretical terms, the cyclical behavior of power markets based on the investors' propensity to defer projects when facing uncertainty. However, it is worth to acknowledge that significant modeling efforts have been done for assessing the long-run behavior of the power industry (Ventosa *et al.*, 2005). Several works have focused on including some behavioral aspects of investors in long-term power market models. Notwithstanding, the methods proposed are based on simplifying the risk-averse profile that defines the investors' response, by adjusting their expectations upon profitability according to predefined patterns. Thus, it is deemed that the literature can be enhanced by including the behavioral nature driving the adequacy of capacity in current power markets.

Given the several causes alleged to originate construction cycles in the deregulated power industry (*e.g.* information asymmetry, risk-aversion, herding behavior, bounded rational expectations and strategic behavior), this paper focuses on investigating the relationship between such cycles and the postponement of investment decisions under uncertainty. For this purpose, a novel framework for valuing generation projects under uncertainty is integrated with a long-term electricity market model. The formulation of a competitive generation system is based on Olsina *et al.* (2006), since it is recognized for describing the dynamics of capacity adequacy by following System Dynamics (SD) simulation approach. However, this research is different as it centers on modeling the microeconomics of investors' decision-making based upon profitability expectations under uncertainty. In that sense, here it is considered that the construction of new power plants is a function of the strategic flexibility given by the postponement option under uncertainty. Therefore, Real Options Analysis (ROA) is applied to derive an optimal investment policy by weighing the value of exercising investments immediately and waiting for more favorable conditions.

The remainder of this work is organized as follows. Section 2 introduces the methodological framework of the proposed investment valuation method. In Section 3, main outlines on the long-term dynamic market model, the description of uncertainties, the formation of expectations upon profitability, and the decision-making process are provided. Simulation results, as well as the validation process of the proposed model, are presented in Section 4. Finally, Section 5 offers concluding remarks and prospects for further research.

2 Methodological framework

2.1 State-of-the-art review

In the context of the present study, the model of a liberalized electricity market is used for gaining insights about the long-term evolution of its structural parameters, namely the installed capacity. Since the addition of new power plants now involves multiple, self-oriented firms, it is essential that the model integrate the logic behind their autonomous decision-making. Several modeling approaches are suitable for describing the long-run behavior of the deregulated industry, from a financial point of view (Sterman, 1991). In particular, it has been found that *simulation models* are suitable for capturing actual behavioral features of investors in power markets, such as bounded rationality, learning abilities, imperfect foresight, etc. (Ventosa *et al*., 2005). In that context, System Dynamics (SD) is a modeling approach with a vast literature body about the development of simulation models of complex systems (Baum *et al.*, 2015). SD has been used widely during the last decade for addressing the description of the long-term development of electricity markets, though is regaining interest in recent years (Leopold, 2015; Ahmad *et al.*, 2016; Rios *et al.*, 2016). The SD approach focuses on identifying the feedback structure of a complex system, at a macroscopic level, and the logical interrelationships among its components. Then, it aims to deliver a dynamic response in the long term by solving the governing non-linear differential equations. A well-founded background on this modeling approach can be found in Sterman (2000).

Generally, dynamic models are well-known for suggesting a volatile long-term behavior of the deregulated power sector. The situation is explained due to the inherently unstable interaction between the power exchange and the profitability expectation of investors. In order to gain insights about this complex interaction, SD provides a tool known as the Causal Loop Diagram (CLD[2]), which helps in giving a perspective about the feedback structure of the system under analysis. Such perspective eventually allows formulating the differential equations that rigorously describe the long-term system dynamics. The literature contains an example of the feedback structure that formalizes the process of capacity expansion in this study context through a CLD (Olsina *et al.*, 2006). Unlike in the centralized paradigm, here a delay representing the investors' decision-making under uncertainties is one of the hypothesized factors preventing the timely adequacy of the installed capacity. This delay denotes the *decision time* necessary for investors to develop enough certainty about the recovery of capital costs. Since investments in power plants are no longer of the *now-or-never* type, investors are then likely to wait for the arrival of new and better information (though never complete) before undertaking new investment projects.

With the advent of deregulation of the power industry, the decision-making of new generation investments has come to depend upon profitability expectations. In that context, the prevailing market design has been the *energy-only market* (*e.g.*, Bunn and Larsen, 1992; Bunn and Larsen, 1994; Kadoya *et al.*, 2005; Eager *et al.*, 2010; Pereira and Saraiva, 2011; Osorio and van Ackere, 2016; Movahednasab *et al.*, 2017). In addition, many works have discussed alternatives for remunerating the generating capacity after acknowledging the existence of imperfections and market flaws perceived in real markets (*e.g.*, Vázquez *et al.*, 2002; Neuhoff and de Vries, 2004; Batlle and Rodilla, 2010; Olsina *et al.*, 2014). Most of the revised SD models have assessed the implementation of mechanisms such as the so-called *capacity payments* and *capacity markets* (*e.g.*, Ford, 1999; Assili *et al.*, 2008; de Vries and Heijnen, 2008; Hasani and Hosseini, 2011; Pereira and Saraiva, 2013; Hary *et al.*, 2016; Ibanez-Lopez *et al.*, 2017).

According to the literature, an additional capacity remuneration mechanism can be either fixed or dynamic. In addition, it can be classified as a price or quantity-based mechanism (Olsina *et al.*, 2014). An example of price-based dynamic remuneration is the mechanism introduced in England and Wales between 1990 and 2001. Under this

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[2] The CLD is a tool from SD modeling, useful to depict the feedback structure of systems. A CLD consists of variables connected by arrows denoting causal influence among variables. Each causal link is assigned a polarity (either positive (+) or negative (–)), indicating how the dependent variable changes when so does the independent variable.

scheme, generators received a marginal clearing price in addition to a price uplift given by the probability of capacity shortfall, equal to the Loss of Load Probability (LOLP), times the power scarcity price, given by the Value of Lost Load (VOLL). A quantity-based method for rewarding generating capacity involves a capacity market juxtaposed to the conventional energy-only market. Here, an obligation of capacity is computed in advance, and it equals a peak demand forecast plus a target reserve margin. Generators make bids of existing and new capacity seeking to fulfill the obligation. Then, the clearing price set in the capacity market is used to derive an additional remuneration for investors. This design is now operative in France and in Great-Britain (RTE, 2014; DECC, 2014).

Despite the general agreement on the market dynamics, the prevailing modeling design still assumes a risk-neutral profile for investors. Therefore, so far only a few long-term models have characterized the risk-aversion of investors when deciding the addition of new capacity. Some examples incorporate an Internal Rate of Return (IRR) delayed by a fixed investment time, which denotes the time necessary for developing enough certainty about the project feasibility (*e.g.*, Olsina *et al.*, 2006; Olsina and Garcés, 2008). In the paper by Sánchez *et al.* (2008), the viability of new power plants is based on a minimum rate of return, which denotes the cost of debt incurred by the generating company obtained by applying concepts of credit-risk theory. Other works focus on adjusting the investor's previous risk-neutral expectations. For instance, Eager *et al.* (2012) include the Value at Risk (VaR) in the definition of project profitability. Moreover, Abani *et al.* (2016) expand the previous concept by including the Conditional Value at Risk (CVaR) for correcting the Net Present Value (NPV) of new power plants. Finally, Petitet (2016) and Petitet *et al.* (2017) propose a concave utility function for representing the value of the project under a risk-aversion assumption.

2.2 Contribution of this work

Few works have focused on representing the risk-averse profile of investors in long-term electricity market models. The revised methods have mainly based on adjusting the profitability expectations with the objective of accounting for the risk-averse response of investors. Despite these efforts, it is deemed that the literature can be expanded in order to describe further behavioral features governing the capacity adequacy in actual power markets. In fact, theoretical and empirical evidence suggests that investors are likely to defer new projects under uncertainty about future cash flows (Dixit and Pindyck, 2012; Arango and Larsen, 2011). This implies that the value of strategic flexibility for seizing opportunities and cutting losses contingent upon market evolution is, at least intuitively, accounted for (Blanco and Olsina, 2011). In that sense, the quantification of strategic flexibility involves a risk management technique, suitable for coping with major market uncertainties in order to achieve a timely investment execution.

The quantification of investment strategic flexibility is strongly associated to the concept of Real Options Analysis (ROA). ROA provides a well-founded background for valuing flexible investments under uncertainty, based on the theory of Financial Options. Unlike the traditional NPV approach to project appraisal, ROA allows valuing opportunities to collect extraordinary profits, inherent to these high-risk projects. For this purpose, the key issue is to use the available options in order to define a lower limit to potential losses while the opportunity of extraordinary profits remains open. In that sense, ROA allows strategically managing a portfolio that includes the underlying project together with all available options. Thus, the availability of these options will usually influence the actual decision-making process, and consequently, must be fairly quantified (Olafsson, 2003). For instance, real options include the chance to postpone, abandon, expand, reduce, switch business, or temporary close and then reopen an investment project (Copeland and Antikarov, 2003).

In that context, the main contribution of this paper is the integration of a long-run power market model with a decision-making framework of generation investments that accounts for the value of strategic flexibility to manage uncertainty. Without losing generality, the work is delimited to compute the value inherent to the option to postpone new power plant projects. Such option refers to an owner's right to defer the project execution while waiting for upcoming (though never complete) information about the market evolution. Value of options embedded in investments in real assets can be computed by means of stochastic dynamic programming (Trigeorgis, 1996). Thus, the proposed approach defines the addition of new units through an investment function that dynamically computes the profitability of exercising new projects immediately and of waiting for more favorable conditions under expectations of uncertain market evolution.

This contribution aims to describe the dynamics of power investments and capacity adequacy in a more realistic fashion, and therefore shed light on the underlying reasons that drive the long-term market development.

3 Modeling decision-making in long-term electricity market models

3.1 Model overview

Real Option Analysis (ROA) is introduced in the long-term market model for describing the decision-making process of investors when considering the addition of new power capacity. Uncertain future electricity prices are endogenous results from the proposed System Dynamics (SD) model, upon which option-based valuations for deciding capacity investments are computed at each time step. Investment decisions in new generating units in turn determine the paths of future power prices in the marketplace. It is worth to note that, in addition to endogenous decisions, the electricity market is driven in the long-term by exogenous random variables, such as demand growth, fuel prices, interest rates, etc. Here, only two external source of uncertainty are considered, *i.e.* fuel prices and demand growth. Appropriate stochastic processes may describe the random fluctuations of these external variables, which perturb the dynamics of power market and introduce uncertainty on the future development of electricity prices. In that sense, the modeling of stochastic variables is detailed in Section 3.3.

Fig. 1 shows a Causal-loop Diagram (CLD) explaining the feedback structure that drives the long-term development of electricity markets under the decision-making framework proposed in this paper. Regarding the logic of power prices, investors are able to assess an instant price signal, based on the state of installed capacity and observations of power demand and fuel costs (loop B1). At the same time, they form expectations upon future prices based on the uncertainty driving the evolution of the same parameters (loop B2). ROA is integrated to the dynamic market model through a technique based on stochastic dynamic programming. This method allows deriving the Exercise Value (EV) and the Continuation Value (CV) that are used for guiding the decision-making of new power plants. First, the CV gives the expected present value of new projects if the decision is to postpone them. In that sense, the CV is related to the stochastic sample of price signals that accounts for investors' expectations upon uncertain market conditions at some future time. Such time is so-called the *Option Maturity*, and represents the moment when the project, if deferred, must be decided (or not) in the future. Second, the EV denotes the present value of undertaking new projects immediately. The EV is then associated to the price signal observed by investors according to current market conditions. Then, the decision to invest in new power plants is defined by a Profitability Index (PI), which results from the ratio between the EV and the CV. This index determines the amount of capacity that is added into the system at each simulation step.

Fig. 1

A Stock-and-Flow Structure (SFS[3]) involving the power capacity is embedded into the CLD introduced by Figure 1. Such structure allows denoting explicitly the variables that control rates of flows into stocks. By means of a SFS, the attributes of the capacity from several available technologies can be disaggregated in terms of lifetimes, construction lead-times, etc. In order to apply the proposed investment valuation framework, this paper therefore adopts a SD modeling approach based on a SFS for representing the various system components. This model is inspired by Olsina *et al.* (2006), since it is recognized for providing a broad mathematical formulation of the long-run dynamics of liberalized electricity markets. The literature already contains examples that followed such work (*e.g.*, Assili, *et al.*, 2008; Olsina and Garcés, 2008; Hasani and Hosseini, 2011). In the following, main outlines of the referred market model are presented.

3.2 Modeling long-term market dynamics

The long-term evolution of electricity markets is mainly driven by movements of the Aggregate Supply Curve (ASC)[4]. Thus, a test power system organized under an energy-only market design is considered. Additional remuneration mechanisms are noteworthy

[3] The SFS is a tool from SD modeling, useful to characterize the state variables of the system and to generate information upon which decisions and actions are based. A Stock creates delays by accumulating the difference between the inflow and the outflow from a process.

[4] The Aggregate Supply Curve results from accumulating the available system capacity according to an *economic dispatch merit order*, that is, a ranking of available generating units according to their marginal production cost, from lowest to highest.

in the current debate on market design of power systems. However, such discussion is out of the scope of this work.

For the sake of simplicity, the stylized test system considers only a thermal generation system with three conventional technologies: base (coal-fired plants – HC), middle (gas-fired combined cycles – CC), and peak (gas turbines – GT). The initial system capacity sums 16.46 GW, from which 72.31% belongs to HC, 14.43% to CC, and 12.26% to GT. These percentages denote the optimal technology mix, given the initial conditions of load curve and annual durations that involve the dispatch of generation units from each technology in order to serve load at minimum cost. In that sense, the duration at which the cost of using two technologies turns equal is obtained from the screening curve[5] of the technologies under study. In this model, the average fixed costs for each technology is derived by transforming the investment costs into a payment stream constant over an amortization time. Likewise, variable costs depend on the marginal generation costs for each technology, which result from the product[6] between the average fuel consumption to produce an energy unit and the fuel price. Typical values assumed for these parameters are provided in Section 4. In addition, the microeconomic foundations that support the calculations can be found in Olsina *et al.* (2006).

The initial capacity includes a reserve margin of 9.75%, which is defined as economically optimal for this test system and thus is based on an optimal level of supply reliability. This level occurs when the cost of serving an additional MW of demand equals the cost of installing and operating an additional MW of peak capacity. From this premise, given the Value of Lost Load (VOLL) and the fixed and variable costs of a MW of peak capacity, the economically optimal duration of load curtailment is also obtained from the screening curve of technologies (Stoft, 2002). Then, by knowing the optimal load shedding duration, the optimal reserve margin is readily derived thanks to a probabilistic reliability model that relates various reserve margins with their expected deficit durations. For each margin, the expected duration of load curtailment is mainly computed from the convolution of the probability of outage of

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[5] Screening curves plot the total cost (fixed plus variable) of using a capacity unit of each technology as a function of the capacity factor.

[6] The product is valid if the generators' heat input functions can be linearized at maximum capacity through the origin.

generating units and the duration defined by the intersection of the available system capacity with the load curve. For simplicity, this involves developing a two-state probabilistic generation model that supposes identical units to allow the calculation of the outage probability table through the binomial distribution. Therefore, an average size and forced outage rate are assumed for all generating units. In this case, parameters are also provided in extent in the Section 4. Moreover, the mathematical background adopted for this simple probabilistic reliability model can also be found in Olsina *et al.* (2006).

The present market model implements a power capacity aging chain with the objective of reproducing the development of the age structure of the test system. Thus, the evolution of the supply curve is determined endogenously as new capacity with higher thermal efficiency is added and old, inefficient capacity is decommissioned. The generating system is differentiated for each technology in five vintages, n_v, which differentiate the capacity productivity in terms of the thermal efficiency of generating units. An estimation of efficiency progression for the generating technologies under study is presented both in Fig. 2 (graphically) and in the Appendix (analytically). The capacity is assumed to remain in the system until the end of its lifetime, which is supposed constant for the simulation period. Therefore, a unit of capacity will reside in each vintage a number of years equal to a fifth of its lifetime. Typical values of 40, 30 and 20 years are adopted for the lifetime of HC, CC and GT capacity, respectively. A different lifetime for each technology is likely as the usage and the number of starts vary significantly among base, middle and peak units.

Fig. 2

The stock of capacity of technology i at any time t is described through an accumulation resulting from the rate at which new capacity enters the first vintage, and the rate at which old capacity abandons the last vintage. Formally, this accumulation is represented by:

$$K_i(t) = \int_0^t \left(\dot{K}_{i1}^{in}(\tau) - \dot{K}_{i5}^{out}(\tau) \right) d\tau + K_i(0) \tag{1}$$

Here, $K_i(0)$ is the initial capacity of technology i, $\dot{K}_{i1}^{in}(t)$, represents the rate at which units are being brought online; and $\dot{K}_{i5}^{out}(t) = \dot{K}_{i1}^{in}(t - T_i)$ is the decommissioning rate, which equal the addition rate at time $t - T_i$, with T_i being the average lifetime of technology i. If Eq. (1) is differentiated by time, the net change in capacity for technology i at any time is expressed by:

$$\dot{K}_i(t) = \dot{K}_{i1}^{in}(t) - \dot{K}_{i5}^{out}(t) \qquad (2)$$

Likewise, the capacity under construction for technology i is derived instantly as an accumulation of the rate at which new power plants are being decided minus the rate at which generating units are being completed:

$$K_i^{UC}(t) = \int_0^t \left(\dot{I}_i(\tau) - \dot{K}_{i1}^{in}(\tau)\right) d\tau + K_i^{UC}(0) \qquad (3)$$

In Eq. (3), $K_i^{UC}(0)$ is the initial capacity under construction for technology i; $\dot{I}_i(t)$ denotes the investment rate; and $\dot{K}_{i1}^{in}(\tau)$ represents the completion rate. It is deemed that $\dot{K}_{i1}^{in}(t)$ depends on the investment rate that prevailed at time $t - \bar{T}_i^C$, $\dot{I}_i\left(t - \bar{T}_i^C\right)$, with $\bar{T}_i^C$ defining the mean construction time for technology i. The construction time differs considerably among generating technologies. In this case, illustrative values equal to 36, 18 and 9 months are assumed for the mean construction time of HC, CC, and GT power plants, respectively. The investment rate at time $t - \bar{T}_i^C$, $\dot{I}_i\left(t - \bar{T}_i^C\right)$, is computed by:

$$\dot{K}_{i1}^{in}(t) = \dot{I}_i\left(t - \bar{T}_i^C\right) = im_i\left(PI_i\left(t - \bar{T}_i^C\right)\right) \cdot \dot{I}_i^{ref}\left(t - \bar{T}_i^C\right) \qquad (4)$$

Here, $\dot{I}_i^{ref}\left(t - \bar{T}_i^C\right)$ is the reference investment rate in technology i for holding the system in the long-run equilibrium, which means, investments made under zero profit expectations. It is expressed as the capacity decommissioning rate, $\dot{K}_{i5}^{out}\left(t - \bar{T}_i^C\right)$, plus the addition rate necessary to cover the expected growth of maximum load served by such technology under an optimal generation mix, $\dot{L}_i\left(t - \bar{T}_i^C\right)$:

$$\dot{I}_i^{ref}\left(t - \bar{T}_i^C\right) = \dot{K}_{i5}^{out}\left(t - \bar{T}_i^C\right) + \dot{L}_i\left(t - \bar{T}_i^C\right) \qquad (5)$$

The mathematical framework for computing the addition rate necessary to cover the expected growth of peak load served by each technology under an optimal generation mix can be seen in Olsina *et al.* (2006). Next, the investment multiplier for technology i, $im_i\left(PI_i\left(t-\bar{T}_i^C\right)\right)$, depends upon profitability expectations formed at time $t-\bar{T}_i^C$. These profitability expectations are synthesized by means of a profitability index, $PI_i\left(t-\bar{T}_i^C\right)$, which is described properly in Section 3.5. As long as the PI increases, more projects based on such technology become profitable, even for projects riskier than average or for companies facing higher firm–specific risks. The amount of information regarding investment plans from competitors and the number of participants following the actions of the leading firms, *i.e.* herding behavior, is critical to the extent to which the investment rate increases with the perceived profitability. Nevertheless, participants are aware of the potential danger that might result from a wave of massive entries when the attractiveness for investing is high. It seems thus logical that the investment responsiveness shows a saturation level for a somewhat high profitability level. Logistic functions (Fig. 3) are adopted in order to capture the effect of such index on the multiplier of the investment rate for each technology, $im_i\left(t-\bar{T}_i^C\right)$. The functions are obtained from the following expression:

Fig. 3

$$im_i\left(t-\bar{T}_i^C\right)=\frac{im_i^{max}}{1+e^{-\left(\alpha_i\cdot PI_i\left(t-\bar{T}_i^C\right)+\beta_i\right)}} \qquad (6)$$

In Eq. (6), for each technology i, im_i^{max} is the saturation level, α_i controls the slope, and β_i determines the location of the function respect to the x-axis. In each case, the tipping point is given when the PI equals one, and thus the investment rate adopts its reference value. The parameters for each technology are included in the Section 4. Following the work by Olsina *et al.* (2006), a different saturation level is assigned. The saturation level for HC power plants is set relatively low, as it is unlikely to expect an over-reaction of investors in this technology. On the opposite, the saturation level for CC units is set relatively high, since a high degree of responsiveness to the profitability level has been observed for this technology in actual markets. Investments in GT share many features with CC projects but, as the own entry might undermine the potential profit and the fact that peak units only profit from very rare events, its investment saturation level might be significantly lower than for CC, however, probably higher than the HC-level.

The average efficiency of the generation system evolves according to the development of capacity in each vintage and the technological progress in thermal efficiencies of new capacity entering the market. Thus, the average efficiency for vintage j of technology i at any time t, $\bar{\eta}_{ij}(t)$, results from the ratio between the accumulation of change in fuel consumption and the existing capacity. This is expressed by:

$$\frac{1}{\bar{\eta}_{ij}(t)} = \frac{1}{K_{ij}(t)} \int_0^t \left(\frac{\dot{K}_{ij}^{in}(t)}{\eta_{ij}^{in}(t)} - \frac{\dot{K}_{ij}^{out}(t)}{\eta_{ij}^{out}(t)} \right) dt + \frac{1}{\bar{\eta}_{ij}(0)} \tag{7}$$

where $\dot{K}_{ij}^{in}(t)$ and $\eta_{ij}^{in}(t)$, and $\dot{K}_{ij}^{out}(t)$ and $\eta_{ij}^{out}(t)$, represent, respectively, the rates and the efficiencies of the capacity entering and abandoning the vintage j of technology i at time t; while $K_{ij}(t)$ represents the residing capacity; and $\bar{\eta}_{ij}(0)$, accounts for the average initial thermal efficiency. In this case, the average initial efficiency for each vintage is determined from the curves presented in Fig. 2, jointly with the age structure of the generation system at the beginning of simulations. In addition, these curves are used to define the efficiency of the capacity being added to as well as being retired from the system over the entire simulation period.

Finally, the average marginal cost of generation for the capacity of vintage j from technology i at any time t, $\overline{MC}_{ij}(t)$, is derived by:

$$\overline{MC}_{ij}(t) = \frac{FP_i(t)}{\bar{\eta}_{ij}(t)} \tag{8}$$

In Eq. (8), $FP_i(t)$ denotes the fuel price, and $\bar{\eta}_{ij}(t)$, the average thermal efficiency for vintage j of technology i at time t.

It is worth to recognize the importance of further emission-free generating technologies in electricity markets worldwide, among which nuclear and hydropower generation are exemplary. Moreover, the mainstream academic discussion now involves the transition towards the large integration of non-conventional renewable technologies, such as wind, solar, etc. In those cases, an additional uncertainty source arises in terms of the availability of the primary energy resource (*e.g.* water inflows). However, it is also argued that fossil fuels would prevail as the world's primary energy source, even in the long run. In fact, as exposed by Covert *et al.* (2016), the International Energy Agency (IEA) estimates that fossil fuels would still supply 79% of the global energy in 2040, if strong policies regarding carbon emissions are not applied (IEA, 2015). In that context, the scope of this article is delimited to shed light on factors driving the market according to the prevailing energy mix.

3.3 Modeling stochastic exogenous market variables

This paper assumes that the market is driven exogenously by stochastic demand and fuel prices. Jointly with the state of installed capacity and fuel consumption, the computation of these variables is essential when modeling the profitability expectations of investors. Here, the prediction of short-term movements of electricity prices provides information to a small extent. In that sense, it is acceptable some loss of chronological information with the objective of gaining in model simplicity. For this reason, demand is characterized by a Load Duration Curve (LDC[7]), which adopts a linear shape in order to avoid unnecessarily complex calculations. Moreover, by neglecting plausible structural changes, it is supposed that the LDC holds its linear pattern over the entire simulation horizon. Therefore, at the start of simulations, the LDC is analytically expressed by:

$$L(0) = \left(L_{min}(0) - L_{max}(0)\right) \cdot d + L_{max}(0) \tag{9}$$

where $L(0)$ is the load level at $t = 0$ exceeding a cumulated duration d, given an initial maximum and minimum demand, $L_{max}(0)$ and $L_{min}(0)$, respectively. In this case, the initial maximum and minimum load equal 15 GW and 10 GW, respectively.

A deterministic exogenous growth rate of 1 %/year is chosen for simulating the demand observed by investors at any time. Thus, known the demand at the start of simulations, the load at time t is given by:

[7] The LDC results from sorting the chronological demand of a certain period (in this case, equal to one year) from higher to lower.

$$L(t) = (L_{min}(0) \cdot e^{g_m \cdot t} - L_{max}(0) \cdot e^{g_M \cdot t}) \cdot d + L_{max}(0) \cdot e^{g_M \cdot t} \qquad (10)$$

Here, g_m and g_M are growth rates of the minimum and maximum demand, respectively. For a linear LDC, it can be demonstrated that if $g_M = g_m = g_L$ the growth of the peak load is equal to the load of the annual energy consumption. In addition, it is worth to mention that exponential functions describe the growth of both variables because here the nature of simulations is of continuous time.

By following the same premise, deterministic exogenous growth rates with typical values equal to 0.02 %/year are used for simulating the prices of hard-coal and natural gas that are observed by investors at any time (Frydenberg *et al.*, 2014). In this case, the initial values for both variables are 6.50 €/MWh and 10.50 €/MWh, respectively. Therefore, known the fuel price at the start of simulations, the fuel price for each technology i at any time t is expressed by:

$$FP_i(t) = FP_i(0) \cdot e^{g_{FP_i} \cdot t} \qquad (11)$$

In Eq. (11), $FP_i(0)$ denotes the fuel price for technology i at the beginning, and g_{FP_i} represents its long-term growth rate.

Rigorously, the growth of power demand and fuel prices is stochastic by nature. In fact, despite being able to observe the key market variables, investors might as well expect future random deviations of the growth rates around their long-term "normal" value. Such deviations might occur due to temporary changes of weather, together with the transitory acceleration and deceleration of the economic activity.

There is a rich literature body documenting research on the stochastic behavior of commodity price movements, such as fuel prices affecting the long-term development of electricity markets. In that sense, yet there is not a broad consensus on the random process that best fits the observed fluctuations of different commodity prices. The selection depends on the commodity itself, the market where the commodity is traded, the length of the data set, the time resolution of price time series (hourly, daily, monthly, etc.), etc. Modelers should be cautious and test this hypothesis for any particular market under consideration.

Notwithstanding, the mean-reverting stochastic process is well-known as a plausible and realistic way to describe uncertain growth rates of commodities, including fuel prices. Indeed, mean-reverting stochastic processes have long been proposed for properly capturing the long-term stochastic dynamics of fuel prices (oil, gas and coal) (Pindyck, 1999). For instance, the article by Schwartz (1997) widely discusses on general economic grounds why mean reversion represents a reasonable and realistic stochastic model for commodity prices. In fact, oil prices are used in that work for statistically testing the mean-reversion hypothesis.

Evidence of mean reversion in spot prices of electricity has also been reported (Pilipovic, 1998; Lucia and Schwartz, 2002; Weber, 2005). In fact, according to Pilipovic (1998), risk management in energy markets requires mean-reverting models as they do the best job in capturing the distribution of energy prices. Currently, mean-reverting specifications are a standard approach used by practitioners for modeling uncertainty upon future and spot prices of energy commodities (Ronn, 2002; Skorodumov, 2008). Moreover, fuel prices following stochastic processes with mean-reversion features, such as the simplest Ornstein-Uhlenbeck (OU) process, are often prescribed in the context of Real Option valuation of power plant investments (Abadie and Chamorro, 2008; Bannör et al., 2016).

Regarding power price models, simple mean-reversion models have also been proposed for capturing stochastic short-term deviations of electricity demand (Barlow, 2002; Kanamura and Ohashi, 2008). In the long-term, growth rates of electricity demand exhibit significant uncertainty and themselves are best depicted as a random variable. In that context, consumption growth is related to population growth and the expansion rate of the Gross Domestic Product (GDP). Evidence of mean reversion is also found with respect to the evolution of the GDP[8] (Mayoral, 2006). This means that economic growth presents unpredictable deviations from the long-term structural mean rate, *i.e.* a "normal" or equilibrium long-term level. These stochastic movements are explained by

[8] Evidence of mean-reversion is found regarding other economic variables as well, including stock prices (Poterba and Summers, 1988) and interest rates (Wu and Chen, 2001)

the arrival of shocks and the occurrence of short-lived endogenous changes in the economy. In fact, recurrence of economic downturns, recessions, recovery and expansions are cyclical patterns displayed by most economies. The probability of a reversion to the normal growth rate is higher if deviations of GDP from the mean are large.

As aforementioned, the stochastic changes in economic output are conveyed to electricity consumption. Empirically, it has been observed that the growth rate of electricity consumption typically declines earlier than a recession is confirmed and usually resume before economic recovery can be measured. The dependence of changes in electricity consumption upon fluctuations of the economic output is supported by significant evidence from countries worldwide (Ghosh, 2002; Jumbe, 2004; Yoo, 2005; Apergis and Payne, 2009; Ouédraogo, 2010; Ciarreta and Zarraga, 2010; Shahbaz et al., 2011; Gurgul and Lach, 2012; Omri, 2013; Wolde-Rufael, 2014).

In that sense, Olsina (2005) have estimated parameters of OU processes with actual data on power consumption growth rates of three different power systems (Argentina, Spain and Germany). Simulated paths with the adjusted OU models replicate well the stochastic changes in consumption growth rates observed in historical datasets. Based on that empirical research, OU processes have been used to describe stochastic demand growth rates in further works embodying the state-of-the-art on the assessment of the long-run investment dynamics of liberalized power markets (Pereira and Saraiva, 2011; Hasani and Hosseini, 2011; Pereira and Saraiva, 2013).

Hence, it is assumed that this stochastic model is realistic enough in order to demonstrate the practicability of integrating ROA in the long-term power market, given uncertainty in fuel prices and electricity demand. The mean-reverting process thus involves an uncertain variable that evolves fluctuating around a known mean. Mathematically, the common mean-reverting process, known as the *arithmetic Ornstein-Uhlenbeck stochastic process*, is given by (Gillespie, 1996):

$$dg = \eta \cdot (\bar{g} - g) \cdot dt + \sigma \cdot dz \tag{12}$$

Here, the expected change in a growth rate, dg, after a time increment, dt, depends upon the deviation of a growth rate, g, from its long-term value, $\bar{g}$, and a speed of the reversion towards the mean, η. In addition, it depends upon a volatility parameter, σ, and a variable following a *Wiener process*, also known as *Brownian Motion*, dz. It can be shown that an infinitesimal increment of the *Wiener process*, dz, can be denoted in continuous time by:

$$dz = \varepsilon \cdot \sqrt{dt} \tag{13}$$

where ε denotes one realization for a normally distributed random variable with mean zero and standard deviation of one, *i.e.* $\varepsilon = N(0,1)$. In order to represent the uncertainty driving the market evolution in a more realistic way, it is reasonable to assume a correlation between, in one hand, the growth rates of power demand and capacity, and, on the other hand, the growth rates of prices for hard-coal and natural gas. In that sense, the set of N random variables $\varepsilon_n; n = 1,2, \ldots, N$ is replaced by the set of N correlated variables $\theta_n; n = 1,2, \ldots, N$. For computing the values of θ_n, the *Cholesky decomposition* is applied to the *correlation matrix*, B (Huang, 2009; Pringles *et al.*, 2015). This is expressed by:

$$\mathrm{chol}[B] = \mathrm{chol}\begin{bmatrix} \beta_{11} & \cdots & \beta_{1N} \\ \vdots & \ddots & \vdots \\ \beta_{N1} & \cdots & \beta_{NN} \end{bmatrix} = AA^T \tag{14}$$

Here, B is composed by the correlation coefficients between the N variables under study, $\beta_{ij}; i,j = 1,2, \ldots, N$. In that sense, this work assigns a correlation of 0.80 for the growth rates of power demand and capacity, as well as a correlation of 0.70 for the growth rates of prices for hard-coal and natural gas. In addition, it is worth to note that all elements from the diagonal of B equal 1, *i.e.* there is full correlation between one variable and itself. In Eq. (14), A is a lower triangular matrix with elements $\alpha_{ij}; i,j = 1,2, \ldots, N$, while A^T is the transpose matrix of A. Then, the value of $\theta_n; n = 1,2, \ldots, N$ is computed as the linear combination of A, and the vector of independent variables $\varepsilon_n; n = 1,2, \ldots, N$, which size is $N \times 1$:

$$\begin{bmatrix}\theta_1\\ \vdots\\ \theta_N\end{bmatrix}=\begin{bmatrix}1 & \cdots & 0\\ \vdots & \ddots & \vdots\\ \alpha_{ij} & \cdots & 1\end{bmatrix}\times\begin{bmatrix}\varepsilon_1\\ \vdots\\ \varepsilon_N\end{bmatrix} \tag{15}$$

By writing Eq. (12) as a difference equation, Monte Carlo techniques can be applied for simulating multiple stochastic realizations of correlated growth rates under study. Then, a realization r of expected demand and capacity, and fuel prices for both technologies, at some future time $M = t + dt$ is obtained by:

$$L^r(M) = \left(L_{min}(t) \cdot e^{g_L^r \cdot dt} - L_{max}(t) \cdot e^{g_L^r \cdot dt}\right) \cdot d + L_{max}(t) \cdot e^{g_L^r \cdot dt} \tag{16}$$

$$K_T^r(M) = K_T(t) \cdot e^{g_K^r \cdot dt} \tag{17}$$

$$FP_i^r(M) = FP_i(t) \cdot e^{g_{FP_i}^r \cdot dt} \tag{18}$$

where $L^r(M)$ is a realization of the load level at time M exceeding a cumulated duration d, given the peak and minimum demand observed at any time t, $L_{max}(t)$ and $L_{min}(t)$, respectively, and a stochastic growth rate, g_L^r, expected at time M. In Eq. (17), $K_T^r(M)$ denotes a possible evolution for the system installed capacity at time M, and it is based on a realization of the correlated growth rate, g_K^r. In that sense, $K_T(t)$ represents the total system capacity at any time t, which results from the dynamic model presented in the previous subsection. Similarly, $FP_i^r(M)$ is a realization of the fuel price for technology i expected at time M, given the fuel price observed at any time t, $FP_i(t)$, and a stochastic, correlated growth rate, $g_{FP_i}^r$. An example of simulation of the observed peak demand at some time t after the beginning of simulations, alongside with the multiple Monte Carlo realizations derived from it and expected at time M, is illustrated in Fig. 4.

Fig. 4

In this paper, the economical demand for power is considered price-irresponsive in the short-term. This assigns customers with an inability to adjust their consumption at short notice in response to sudden price changes. However, it is assumed that consumers will not decide to purchase any more power if the market price rises above the cost of being curtailed. This cost, represented by the VOLL, is generally administratively set by the authorities in order to contemplate the case when the market cannot be cleared because the available power capacity is insufficient to satisfy the price-inelastic demand. In this case, the VOLL adopts an exemplary value of 1000 €/MWh.

It is worth to recognize that electricity demand, if exposed to spot prices, might develop some degree of elasticity over a longer period by reallocating or reducing consumption. Moreover, the integration of novel technologies[9] offers new possibilities for consumers to be much more responsive to prices. In that context, changes in load patterns might have major impact on the long-run evolution of power markets. However, long-term demand adjustments are not considered since the lack of empirical evidence on the extent that this occurs[10]. Thus, the investigation of price-driven long-run alteration in consumption patterns is beyond the scope of this work.

Notwithstanding, it is important to highlight as well that here expectations upon demand growth, which are based on observations made continuously within the simulation period, follow a mean-reverting stochastic process. The adoption of this process allows denoting some degree of responsiveness, at least indirectly, of the demand growth. For instance, after a decrease in demand growth, which might be seen as a response to rising prices, the process's component of reversion towards the mean will define an increase in the growth rate, which might result from decreasing prices. Thus, the underlying self-balancing mechanism of the mean-reverting process might be as well interpreted as the permanent adjustment of demand growth in response to the long-term movements of power prices.

3.4 Modeling expectations upon profitability

A price duration model is used to derive the market signals for investment decision-making in each technology. In this case, the information on the long-term distribution of power prices is assumed to be accurately represented by an annual Price Duration Curve (PDC). As shown in Fig. 5, each PDC is defined schematically based on the ASC and

[9] For instance, these technologies include, smart grids, distributed generation and energy storage.
[10] A higher responsiveness of consumption to power prices would likely be a major source of complex dynamics in future electricity markets. Dynamic interaction of demand elasticity with capacity investments is a relevant issue deserving further investigation.

the LDC, both of which have been introduced in previous subsections. In that context, the PDC yields (in the x-axis) the yearly probability, D_{ij}, for the capacity of vintage j from technology i to operate with the market price over its own marginal cost of generation (in the y-axis). The set of probabilities for each vintage is then computed by:

Fig. 5

$$D_{ij} = \begin{cases} 1 & q \cdot K_{ij}^{cum} < L_{min} \\ \frac{q \cdot K_{ij}^{cum} - L_{max}}{L_{min} - L_{max}} & L_{min} \leq q \cdot K_{ij}^{cum} \leq L_{max} \\ 0 & q \cdot K_{ij}^{cum} > L_{max} \end{cases} \quad (19)$$

where L_{min} and L_{max} are the peak and maximum demand, respectively, and K_{ij}^{cum} denotes the aggregate system capacity up to vintage j from technology i, according to the *dispatch merit order*. Additionally, q represents a system-wide expected availability of generating units. This coefficient is derived at the initial time from the relation between the available capacity for the entire system, $K_T^{aval}(0)$, and the maximum demand, $L_{max}(0)$:

$$q = \frac{K_T^{aval}(0)}{L_{max}(0)} \quad (20)$$

The available system capacity is attained as the intersection of the deficit duration, related to the probabilistic reliability model mentioned in Section 3.2, with the LDC at the start of simulations. This is expressed by:

$$K_T^{aval}(0) = \big(L_{min}(0) - L_{max}(0)\big) \cdot D_{def}(0) + L_{max}(0) \quad (21)$$

By neglecting non-fuel costs and operational constraints, it is assumed that power plants will be dispatched each time the prevailing price exceeds their generation marginal costs. In that sense, the operating profits that one MW of new capacity from technology i would make on the power market in one year, π_i [€/MW·year], can be determined from the enclosed area between the PDC and the marginal cost of such generation unit, $\overline{MC}_i$ [€/MWh]:

$$\pi_i = 8760 \cdot \int_0^{D_i} [PDC - \overline{MC}_i] \cdot dD \qquad (22)$$

In Eq. (22), 8760 represents the amount of hours in one year, while D_i denotes the annual probability (adimensional) for one MW of new capacity from technology i to operate with the market price over its own marginal costs. Next, by assuming that the operating profits from the first year will remain constant, the present value of the expected stream of operating profits that a new MW from technology i would make over an amortization period $\bar{T}_i^A$, $OP_{\bar{T}_i^A}$ [€/MW], can be approximated as:

$$OP_{\bar{T}_i^A} = \pi_i \cdot (1+\rho)^{-\bar{T}_i^C} \cdot \frac{1}{\rho} \cdot \left[1 - (1+\rho)^{-\bar{T}_i^A}\right] \qquad (23)$$

In Eq. (19), π_i represents the annual unitary rents expected by technology i, and ρ is the required revenue rate by which $OP_{\bar{T}_i^A}$ must be discounted. In this case, constant values equal to 25, 20 and 15 year are considered for the amortization time of HC, CC and GT power plants, respectively. Likewise, a required discount rate of 12.5 %/year is adopted for each technology. Finally, it is deemed that investors account for a time lag in the construction of new power plants. Thus, the present value of expected profits is also discounted over an average construction time for each technology, $\bar{T}_i^C$.

The approximation defined by Eq. (23) can also be understood as an efficient energy forward contract auction. In real markets, these auctions offer long-term contracts based on current price levels, aiming at reducing financial risks for newcomers in the generation activity (Moreno *et al.*, 2010).

3.5 Modeling decision-making under uncertainties

A novel modeling approach is proposed to determine the aggregate addition of capacity into the test generation system. The model intends to reproduce the decision-making of investors in each technology according to profitability expectations under uncertainty, and thus is elaborated upon the notion of ROA. This is the main contribution of this paper, since it focuses on integrating a ROA framework for valuing irreversible investments under uncertainty within a long-term electricity market model. Rigorously, an optimal investment policy for each technology can be derived at any time by comparing the value of immediately undertaking new projects, *i.e.* the EV, with the value of projects if the decision is to postpone them in order to be reassessed in the future, *i.e.* the CV. In that sense, the EV is related to the price signal observed by investors at each simulation step, according to current market conditions. Conversely, the CV is associated to a stochastic sample of price signals that accounts for investors' expectations upon uncertain market conditions at some period after each the simulation step. Such period, so-called the *Option Maturity*, represents the future moment when the project must be decided (or not) if it is deferred.

In both cases, the price signals are given by PDC's. As stated in the previous subsection, each PDC denotes a highly non-linear function that is defined schematically from an ASC and a LDC. Thus, the price signal related to the EV is derived from the ASC and the LDC that consider market conditions observed by investors at each simulation time. Likewise, the sample of price signals associated to the CV depends on stochastic samples of both the ASC and the LDC. The samples represent investors' expectations upon the uncertain evolution of market conditions at the *Option Maturity*. In practice, this assumption seems plausible since investors are able to follow the behavior of variables defining the market price signal. At the same time, they might keep track of the movement of the price signal in recent years and form expectations upon a number of future market movements, based on the observed value and its perceived volatility. This defines the difference between the two sources of information that ultimately are used here to model both, the EV and the CV.

In order to define the ASC and the LDC that are observed by investors first it is required to compute the maximum and minimum demand and the fuel price for each technology at each simulation step. For this purpose, variables are assumed to follow deterministic patterns according to exogenous long-term growth rates, as shown by Eq. (10) and Eq. (11), respectively. It is worth to note that the remaining parameters that determine the observed ASC, *i.e.* the capacity and the thermal efficiency from each vintage of each technology, are state variables and thus result from dynamic simulations. Likewise, for determining the stochastic samples of ASC and LDC that are expected by investors first it is necessary to obtain samples of maximum and minimum demand, capacity and fuel price for each technology at the *Option Maturity*. These variables are derived from the values observed at each simulation step, jointly with stochastic samples for the growth rates expected at the *Option Maturity*, as shown by Eq. (16), Eq. (17) and Eq. (18). Then, the sample growth rates are obtained by applying multiple Monte-Carlo realizations of the *arithmetic Ornstein-Uhlenbeck stochastic process* that determines the uncertain evolution of the variables under consideration (Eq. (12)).

It is important to mention that the major concern of applying different stochastic processes in the scope of ROA is mainly related with the utilization of the binomial lattice approach for option pricing. Under this approach, the uncertain underlying asset must be described according to a lognormal probability distribution. In order to overcome these limitations, this paper does not apply such method, but utilizes a ROA approach based of stochastic dynamic programming. Backward Dynamic Programming based on Expected present value (DPE) involves a suitable optimization technique for computing the EV and the CV for each technology at any time (Blanco *et al.*, 2012). Unlike other dynamic programming tools (*e.g.* the binomial lattice method), the DPE performs particularly well when dealing with highly volatile profits, as in this case. In that context, the decision problem for technology i at the *Option Maturity* $M_i = t + dt_i$ can be modeled as:

$$\text{Exercise, if } \mathbb{E}\left[OP_{\bar{T}_i^A}(M_i)\right] > IC_i(M_i) \tag{24}$$

$$\text{Do not exercise, if } \mathbb{E}\left[OP_{\bar{T}_i^A}(M_i)\right] \leq IC_i(M_i)$$

Thus, the value of the deferral option for technology i at maturity time M_i, $V_i^*(M_i)$ [€/MW], can be defined as:

$$V_i^*(M_i) = \max \quad \left[\left(\mathbb{E}\left[OP_{\bar{T}_i^A}(M_i)\right] - IC_i(M_i)\right); 0\right] \tag{25}$$

In Eq. (25), $\mathbb{E}\left[OP_{\bar{T}_i^A}(M_i)\right]$ represents the expected present value of operating profits for a new MW of technology i, derived from the stochastic sample of price signals expected at M_i, while $IC_i(M_i)$ denotes the required investment costs. The number of Monte Carlo realizations of the stochastic variables is set to 50000, in order to satisfy convergence criteria (sampling error) in the statistical estimation of the expected value of future profits. By considering a single period, dt_i, between the current time and the *Option*

Maturity, the continuation value of the postponement option for technology i at time t, $V_i^{cont}(t)$ [€/MW], *i.e.* the project value if the decision is to postpone its execution, can be expressed by:

$$V_i^{cont}(t) = {V_i^*(M)}\Big/{(1+\varphi)^{dt_i}} \tag{26}$$

This formulation for the continuation value involves European real options. In this case, it is assumed that the period dt_i equals 1 year for each technology. Due to irreversibility, the *Option Maturity* might be different according to the characteristics that differentiate base, middle and peak technologies. However, the value assigned here to the period dt_i for each technology intends only to represent a reasonable amount of time after which it is deemed that firms will be willing to reconsider the investment again, if the decision is to defer it. In addition, the φ in Eq. (26) denotes a risk-free discount rate. According to the DPE method, this parameter can be associated to the required return rate for each technology, ρ, which adjustment will then follow a non-neutral valuation of risk. In that sense, a risk-free discount rate equal to 12.5%/year is adopted for each technology.

Next, the exercise value for technology i, $V_i^{ex}(t)$ [€/MW], can be defined as the NPV of a new MW, according to the system state observed at time t. This can be expressed as:

$$V_i^{ex}(t) = OP_{\bar{T}_i^A}(t) - IC_i(t) \tag{27}$$

where $OP_{\bar{T}_i^A}(t)$ represents the expected present value of operating profits for technology i at time t; and $IC_i(t)$ represents the capital outlay for bringing online a new generation unit from technology i. In this model, typical values equal to 1000 €/kW, 600 €/kW and 300 €/kW correspond to investment costs for HC, CC and GT technologies, respectively. These values are assumed constant over the entire simulation period, and thus are used for computing instantly both the CV and the EV, in order to avoid the introduction of exogenous source of dynamics.

The optimal investment policy for technology i at time t, $V_i^*(t)$ [€/MW], can then be derived from the following optimization problem:

$$V_i^*(t) = \max \left[OP_{\bar{T}_i^A}(t) - I_i(t); {V_i^*(M)}\Big/{(1+\varphi)^{dt_i}} \right] \tag{28}$$

As mentioned by Blanco *et al.* (2012), the relationship given by Eq. (27) allows extending the conventional NPV-based rule for characterizing the feasibility of new projects. In that sense, a new investment decision threshold can be defined as follows: *"At any time t, the decision-maker should not invest in a new project (and wait for reassessing it after a given period dt_i unless the current NPV of the investment portfolio (the Exercise Value) is greater than the Continuation Value"*. Inspired by this concept, a new investment Profitability Index (PI) for technology i at time t can be defined as the ratio resulting from dividing the exercise value, $V_i^{ex}(t)$, by the continuation value, $V_i^{cont}(t)$:

$$PI_i(t) = {V_i^{ex}(t)}\Big/{V_i^{cont}(t)} \tag{29}$$

Finally, the PI is used in Eq. (5) to compute the investment multiplier for each technology. It then determines the aggregate investment rate that defines the adjustment of generating capacity from each technology in the electricity market model discussed in this work. In that sense, it is worth to notice that, under this new framework, the timing of investments as a function of the PI can be described schematically, as in Fig. 6 (Luehrman, 1998). Whenever the exercise value is positive and it exceeds the continuation value, the optimal strategy should be to invest now (Region 1). However, when the exercise value does not exceed the value of the deferral option, the investor would be cautious about the uncertain conditions defining the market evolution and would probably reconsider to invest later (Region 2). It is intuitive to suppose that the project appraisal will be much more pessimistic whenever its instant NPV is negative. In that context, it is natural for each generator to withhold investments until new information about the market evolution arrives (Region 3). Moreover, when the exercise value is negative, and its absolute value even exceeds the continuation value, there would be no incentives to invest whatsoever (Region 4). In that case, the investor could even consider switching of business.

Figure 6

4 Simulation and results

4.1 Initial conditions

Simulations were carried out in order to apply the proposed framework. These simulations were performed on the simplified thermal generation system presented in the last section. At the start of simulations, the test system is set to the long-run economic equilibrium. Input data on the attributes of each generating technology and the functions of investment responsiveness is included in Table I and Table II, respectively. Likewise, parameters for describing the stochastic evolution of maximum and minimum demand, total capacity and fuel prices are shown in Table II. The delay differential equations (DDE) defining the market dynamics were solved by means of the dde23 function of MATLAB. The simulation period extends for 20 years.

Table I

Table II

Table III

4.2 Base case simulation

The simulation of installed capacity and reserve margins under the proposed investment valuation framework, alongside with the long-term peak demand, is depicted in Fig. 7. Results are relevant because the proposed approach allows reproducing explicitly the construction cycles that have arisen in several electricity markets after the liberalization (Arango and Larsen, 2011). This is explained due to a more refined characterization of investors' decision-making under uncertainties, in addition to the embedded construction delays for new power plants of each technology.

Fig. 7

The system response is described as follows. Given the zero-profit conditions at the start of simulations, *i.e.* long-term market equilibrium, the continuation value outweighs the exercise value of power plants for each technology. Thus, generators find more attractive to withhold new projects because they have the possibility to invest later and collect extraordinary profits associated to situations of supply deficit (Region 2 and Region 3 in Fig. 6). This leads to a dramatic reduction of reserve margins during the first years, just after the liberalization of the electricity market. Notwithstanding, it is worth to mention that such reduction displays a discontinuous behavior. An explanation is that the completion rate lags the investment rate by the construction time for each technology. Therefore, during the period $t \leq \bar{T}_i^C$, power plants are still being completed according to the ordering rate given under the long-run market equilibrium. Only when $t > \bar{T}_i^C$, the aggregate completion rate start to reflect investment rates resulting from the commercial decisions of investors in each technology. Then, the evolution of installed capacity and reserve margin begins to display a continuous behavior.

The installed capacity decreases until reaching an extremely low value around year 4, when the value of immediately exercising the option to invest finally exceeds the continuation value for each technology. Only then, it becomes attractive to invest due to the high revenues being perceived thanks to the critical supply condition (Region 1 of Fig. 6). A stream of new units is thereby incorporated to the system, and remains until the continuation value begins to surpass the exercise value once again, due to the excess of capacity, around year 14. The construction cycle then starts over once again.

This remarkable fluctuating behavior influences the electricity prices that must be paid by consumers. In Fig. 8, the average annual market price expected under the proposed framework is depicted. According to the alerted cyclical behavior, significant price spikes, coincident with the critical reduction of installed capacity, affect the market. This is a direct consequence of the price model, based on the stepped supply curve and the high value of load curtailment (VOLL). In addition, Fig. 8 shows the average annual cost of production. This is obtained from the weighted average of the marginal cost times the annual probability for the capacity from each vintage to set the market price, obtained through the prevailing Price Duration Curve (PDC) at each simulation step. In this case, the downward trend is explained by means of the steady replacement of old, inefficient generating units by others with higher thermal efficiency. This occurs despite the increase in fuel prices, which grow at extremely low rates, allowing the increase of thermal efficiency to outweigh the effect of the escalation of fuel prices, resulting the net effect in an abatement of the production costs for each vintage over the simulation period. The difference between the patterns of price and production cost reflects the intrinsic impossibility of the system to adjust swiftly the installed capacity to the long-run market development due to the investors' decision-making under uncertainty as well as the embedded construction delays.

Fig. 8

The underlying causes for the simulated long-term market behavior can be appreciated by looking to Fig. 9. This figure shows the capacity under construction over time for each of the considered technologies. In addition, in dashed lines it depicts the capacity under construction required for keeping the system under the long-term equilibrium, *i.e.* when investments are made under zero-profit expectations and thus investment rates exactly offset the decommissioning of old power plants and the long-term growth of demand. In that sense, the capacity under construction for each technology displays a quite volatile pattern under the proposed decision-making framework. By comparing this simulation with the price in Fig. 8, it can be seen that the decisions to invest in new capacity increase when the price escalates and, on the opposite, decrease when the price drop again because of the new generating units being brought online. An explanation for this fluctuating behavior is that investors are reluctant to commission new power plants until perceiving clear and consistent evidence of profitability. This is mainly because of the uncertain expectations upon the market development and the high component of irreversibility in generation investments.

Fig. 9

4.3 Sensitivity analysis

Sensitivity analysis were carried out respect to the long-term demand growth rate and the volatility of demand growth rate expected at the *Option Maturity* for each technology. Fig. 10 depicts the simulation of capacity adequacy with long-term load growth rates of 0.5%/year and 2%/year. For the higher growth rates, it is observed that the system shows a more dramatic reduction of reserve after the start of simulations. An explanation is that increased growth rates yield greater expectations upon deficit conditions in the short term for each technology. Hence, the continuation value severely outweighs the exercise value, and investors constrain even more the addition of new capacity. For the higher growth rates, when investment exercise becomes attractive, it is observed that the stream of new power plants is incorporated to the system at a higher rate. This would cause later a pronounced situation of capacity excess, which would define again the start of a new construction cycle, but of increased amplitude. Therefore, it is reasonable to predict a more volatile market condition as the demand growth rate increases.

Fig. 10

Fig. 11 shows the simulation of capacity adequacy for different volatilities of the demand growth rate expected at the *Option Maturity* for each technology. With higher volatilities, it is found that the market experiences a more dramatic depletion of reserve margins in the first years of simulation. As in the previous example, this is explained because the higher the volatility, the more likely the scarcity conditions that imply extraordinary profits in the short term for each technology.

Fig. 11

After the first drop of reserve margins, the market with lower volatilities of load growth needs to reach an overstepped capacity situation for the continuation value to exceed once again the investment exercise value for each technology. This behavior is explained due to the lack of uncertainties about the market evolution, which gives the signal to invest in more power plants than required. Later on, this leads to a more dramatic reduction of reserve margins, which impacts directly on the stability of market prices. On the opposite, fewer investments are required to be added for the continuation value of each technology to surpass again the exercise value in a highly volatile demand scenario. Despite the more stable market behavior, here the reserve margin is always below the economic optimum, which settles the market-clearing price on a rather high average value. An explanation is that investors are likely to execute new projects proportionally in order to maintain low reserve margins and to secure high scarcity rents, in response to expectations upon a highly uncertain market evolution.

The described patterns are coherent with the experience in actual electricity markets. In fact, lessons learned suggest that the combination of strong demand growth rates and high volatilities was one of the main reasons that led to crises in the supply security of several markets after the deregulation, *e.g.* in South America (Rudnick *et al.*, 2005)

4.4 Formal validation of the System Dynamics model

This section addresses the formal validation of the proposed electricity market model. The procedure follows the implementation guidelines discussed by Barlas (1996), which includes a renowned literature body regarding SD modeling validation. The process thus involves the application of a "minimum" most-crucial set of validation tests, which are classified here according to the next two steps:

1) Direct structure tests
2) Structure-oriented behavior tests

First, *direct structure tests* evaluate the validity of the model structure directly, based on knowledge about the actual structure of the system under study. This needs taking each mathematical equation (or logical relationship) that comprises the model individually, and comparing its suitability with evidence from the real system (Barlas, 1996). Second, *structure-oriented behavior* tests study the structure indirectly, by applying behavior tests on model-generated behavior patterns (Barlas, 1989; Forrester and Senge, 1980). Barlas (1996) highlights the overall importance of this second step because it reveals information on structure adequacy, while holding potential for being quantifiable and formalizable.

Given the crucial nature of structure validity in SD modeling, the emphasis in both steps is on model structure. However, it is worth to recognize the existence of one last step (so-called *behavior pattern tests*), which assesses the prediction accuracy of the model output against real data. The reproduction of the actual behavior of the system under study is a relevant feature for SD models, but meaningful only after enough confidence in the model structure is developed (Barlas, 1996). Moreover, for a significant number of works, including this one, it is not possible to address the third step since they discuss theoretic electricity markets with no historical data to compare with the model response.

4.4.1 Direct structure tests

In this step, it is proposed to apply the *dimensional-consistency* and the *direct extreme-condition tests* (Forrester and Senge, 1980). First, the *dimensional-consistency test* entails checking the right-hand side and left-hand side of each model equation for dimensional consistency. Second, the *direct extreme-condition test* involves evaluating the validity of model equations under extreme conditions, by assessing the plausibility of the resulting values against the knowledge of what would happen under a similar condition in real life. Each equation is thus tested by assigning extreme values to its input variables, and comparing the value of the output variable to what would logically happen in the real system under the same extreme condition. It is worth to mention that these tests do not imply dynamic simulation, since they are applied to each equation in isolation, statically.

Due to space restrictions, results from both tests are not included in this paper. However, documents summarizing such results are available online[11] . The analysis is organized acknowledging the causal interrelationships among model equations. Thus, both tests address the output of model equations sequentially by following the logic of the balancing feedback structure shown in the CLD of Fig. 2.

In particular, the *direct extreme-condition test* account for the initial peak and minimum demand and the financial costs for investors in each generating technology ($L_{max}(0)$, $L_{min}(0)$ and ρ, respectively) as test input variables. It is proposed to evaluate the response of each model equation under three extreme conditions:

1) Large initial maximum and minimum demand
2) Small initial maximum and minimum demand
3) Large financial costs for investors in each generating technology.

[11] For results of the direct extreme-condition tests, please see the file available on https://goo.gl/cQ2vou. For results of the dimensional-consistency tests, please see the file available on https://goo.gl/eKJM9T.

The response is traced through each equation in order to define ultimately the power capacity under construction (K_T^{UC}) of the entire system. The evaluation is performed immediately after the beginning of simulations, *e.g.* at time equal to around 2 months. The extreme values assigned to these variables and the expected ultimate responses are presented in Table IV.

Table IV

In the first condition, an increased demand implies an increase in profit expectations for each generating technology. This is explained because the average market price rises due to the permanent dispatch of the most-expensive generating units and the higher shortfall probability. Ultimately, investment signals for each technology escalate, and thus the system capacity under construction increases respect to the base case. The second condition, however, poses the opposite situation. Here, the market price plummets because the cheapest generating units are sufficient for serving the load. In addition, the probability of load shedding drops due to the large reserve of power capacity. Thus, expectations upon profits for each generating technology decrease and consequently the capacity under construction falls. The situation is quite different for the third condition. Here, no matter what market situation, the increased financial costs for each technology entail a dramatic reduction for the value of the cash flows expected over the amortization period. Therefore, investment signals for each technology fall and the capacity under construction of the entire system collapses. Finally, these results allow concluding that all model equations pass all of the proposed tests.

4.4.2 Structure-oriented behavior tests

In this stage, it is proposed to execute *indirect extreme-condition tests*. Extreme-condition testing implies assigning extreme values to selected parameters and comparing the model-generated behavior to the expected behavior of the real system under the same extreme conditions. In that sense, a formal process inspired by the

Reality Check® functionality of Vensim®[12] is followed. The tests are formalized in terms of two types of equations. First, *Test Input equations* allows specifying the conditions under which a Constraint is binding. Second, *Constraint equations* make statements about consequences that should result from a given set of conditions. They are called Constraints because they specify the way in which Test Inputs should constrain the system behavior. The violation of a Constraint indicates a problem with the model. Thus, the proposed extreme-condition tests to be applied are:

1) First test: Large Demand, Increasing Capacity under construction
2) Second test: Small Demand, Decreasing Capacity under construction
3) Third test: Large financial costs, Decreasing Capacity under construction

First test: Large Demand, Increasing Capacity under construction

The Test Input equations are:

$$L_{max}^{TEST}(0) = L_{max}(0) \times 25 \tag{30}$$

$$L_{min}^{TEST}(0) = L_{min}(0) \times 25 \tag{31}$$

The Constraint equation is:

$$\text{THE CONDITION: } L_{max}^{TEST}(0) = L_{max}(0) \times 25 \text{ AND } L_{min}^{TEST}(0) = L_{min}(0) \times 25$$
$$\text{IMPLIES: } K_T^{UC}(t) > K_T^{UC^*}(t) \tag{32}$$

First, $L_{max}^{TEST}(0)$ and $L_{min}^{TEST}(0)$ are the test initial peak and minimum demand, respectively; while $L_{max}(0)$ and $L_{min}(0)$ are the initial peak and minimum demand for the base case, respectively. Then, $K_T^{UC}(t)$ is the total system capacity under construction at any time $t > 0$ and $K_T^{UC^*}(t)$ is the capacity under construction in the long-term equilibrium.

Second test: Small Demand, Decreasing Capacity under construction

[12] Vensim® is an industrial-strength simulation software for improving the performance of real systems. For more information, please visit: https://goo.gl/ZnE2Kp.

The Test Input equations are:

$$L_{max}^{TEST}(0) = L_{max}(0)/25 \quad (33)$$

$$L_{min}^{TEST}(0) = L_{min}(0)/25 \quad (34)$$

The Constraint equation is:

$$\text{THE CONDITION: } L_{max}^{TEST}(0) = L_{max}(0)/25 \text{ AND } L_{min}^{TEST}(0) = L_{min}(0)/25$$
$$\text{IMPLIES: } K_T^{UC}(t) < K_T^{UC^*}(t) \quad (35)$$

In this set of equations, the variables are identical to that from the first test.

Third test: Large financial costs, Decreasing Capacity under construction

The Test Input equations are:

$$\rho^{TEST} = \rho \times 25 \quad (36)$$

The Constraint equation is:

$$\text{THE CONDITION: } \rho^{TEST} = \rho \times 25$$
$$\text{IMPLIES: } K_T^{UC}(t) < K_T^{UC^*}(t) \quad (37)$$

Here, ρ^{TEST} is the test required revenue rate for investors in each generating technology, while ρ is the base case value for the same variable.

The Constraint defined for each test is explained with the analogous logic exposed in the previous section regarding the outcome of the direct extreme condition tests. In that sense, the first, second and third test presented here are associated to the first, second and third direct extreme-condition tests, respectively. Results from dynamic simulations involving the three indirect extreme-condition tests are depicted in Fig. 12, 13 and 14, respectively. In each case, it is verified that none of the Constraints is violated at any time over the simulation period and thus the model pass all of the proposed tests.

Fig. 12

Fig. 13

Fig. 14

5 Conclusion

This paper proposes a novel decision-making framework to model investment dynamics and long-run capacity adequacy in liberalized power markets. The design of this framework has taken advantage of a well-founded background for describing the long-term market dynamics based on System Dynamics simulation approach. However, this work is different as it focuses on modeling the microeconomics of decision-making of generators, accounting for the option to postpone new power plants projects under uncertainty. In that sense, the integration of a valuation framework of irreversible investments under uncertainty, elaborated upon Real Options Analysis, with a long-term electricity market model is the main contribution of this paper.

Here, Real Options Analysis is integrated into the dynamic market model by means of a technique based on stochastic dynamic programming. This technique allows computing at each simulation step the Exercise Value (EV) and the Continuation Value (CV) that are used for guiding the decision-making of new investments from each technology composing the generation system. First, the CV gives the expected present value of new projects if the decision is to postpone them. The CV is related to a stochastic sample of price signals that accounts for investors' expectations upon uncertain market conditions some period after each the simulation step. Such period, so-called the *Option Maturity*, represents the moment when the project must be decided (or not) in the future if it is postponed. Second, the EV denotes the present value of undertaking new projects immediately. The EV is then associated to the price signal that is observed by investors at each simulation step, according to current market conditions.

Simulation results have shown that with the proposed decision-making model the long-term market evolution is defined by explicit construction cycles. This is consistent with the empirical evidence that have been reported from several electricity markets after deregulation as well as with the outcome of previous works that have modeled the market development by considering alternative investment functions. In that sense, the main contribution of this paper is the reproduction of construction cycles by incorporating a mathematical model that describes the investors' decision-making under uncertainty by accounting for the option to postpone new generating units. Sensitivity analysis regarding some relevant exogenous variables have suggested that the combination of strong demand growth rates with large volatilities would derive in an even more fluctuating evolution of the installed capacity. Finally, the model has been validated by applying a formal procedure according to the scope of System Dynamics modeling approach.

5.1 Critical review of the model

This work considers a test electricity system organized under an energy-only market. However, the importance of additional remuneration mechanisms is noteworthy in the current debate on market designs. In that sense, the analysis of capacity remuneration mechanisms might shed light into the suitability of investment incentives for achieving a more stable development of the liberalized power industry in the long term.

In addition, the scope of this article is delimited to shed light on factors driving the market according to the prevailing energy mix based on thermal technologies. Notwithstanding, it is worth to recognize the relevance of further emission-free generation technologies nowadays, *i.e.* nuclear and hydropower. Moreover, the mainstream academic discussion now involves the transition towards the large integration of non-conventional renewable technologies, such as wind, solar, etc. In those cases, an additional uncertainty source arises in terms of the availability of the primary energy resource. In that sense, it is worth to mention that the analysis and price modeling of a pure thermal generation system is much easier than a hydrothermal power system with significant storage capacity in water reservoirs. Uncertainties associated to the intermittency of primary renewable resources could derive in larger uncertainties regarding demand fluctuations. However, it is worth to remark as well that one of the main contributions of this paper is the validation of the outcome of the dynamic ROA model according to the empirical construction cycles that appeared in the earlier implementation of deregulated power markets. In that sense, the market architecture has also seek to be consistent with those situations where a fuel-based energy-only market was predominant.

Finally, this paper considerers the demand as price-inelastic in the short-term. However, electricity demand, if exposed to spot prices, might develop some degree of elasticity over a longer period by reallocating or reducing consumption. Moreover, novel technologies (*i.e.* smart grids, energy storage) expand the possibilities for consumers to be much more responsive to electricity prices. Thus, it is reasonable to expect an impact on the long-term evolution of power markets due to major shifts in load patterns. This topic should be at the center of interest in the upcoming years.

In that context, the proposed mathematical framework is versatile enough to incorporate several extensions and improvements without much effort. Thus, work delving on dealing with these topics is foreseen in further projects.

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Figures

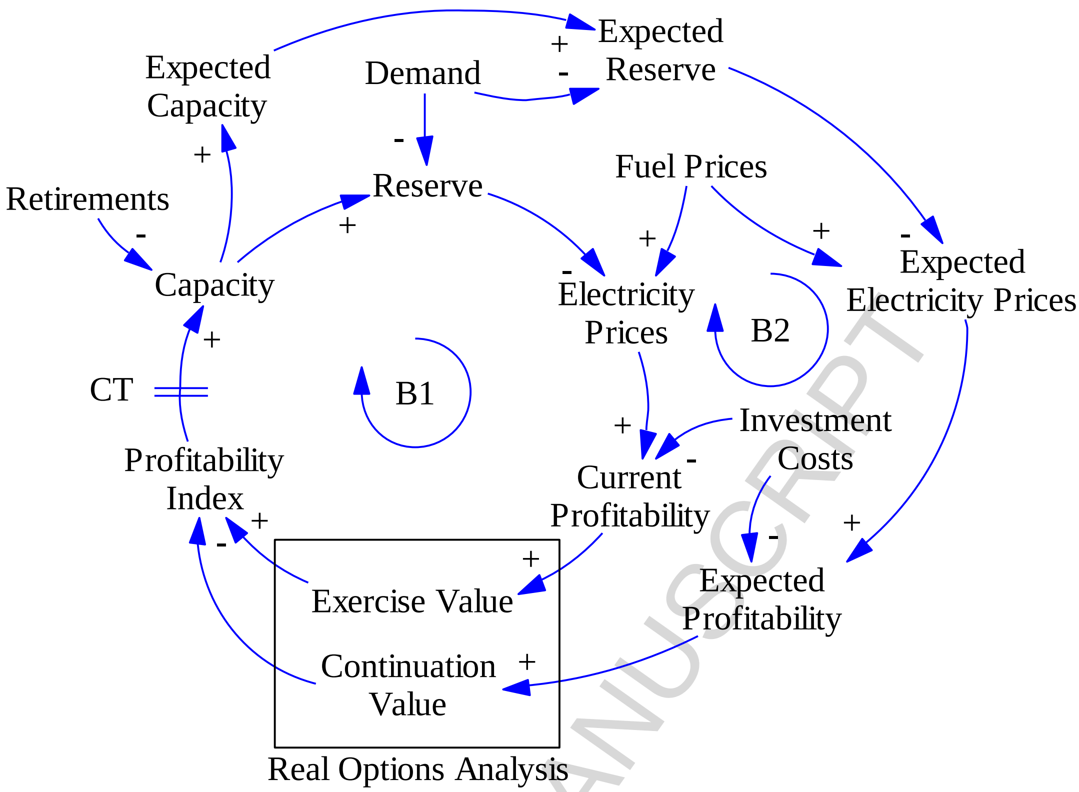

Fig. 1: Causal-loop Diagram of the long-term development of electricity markets under the proposed decision-making framework.

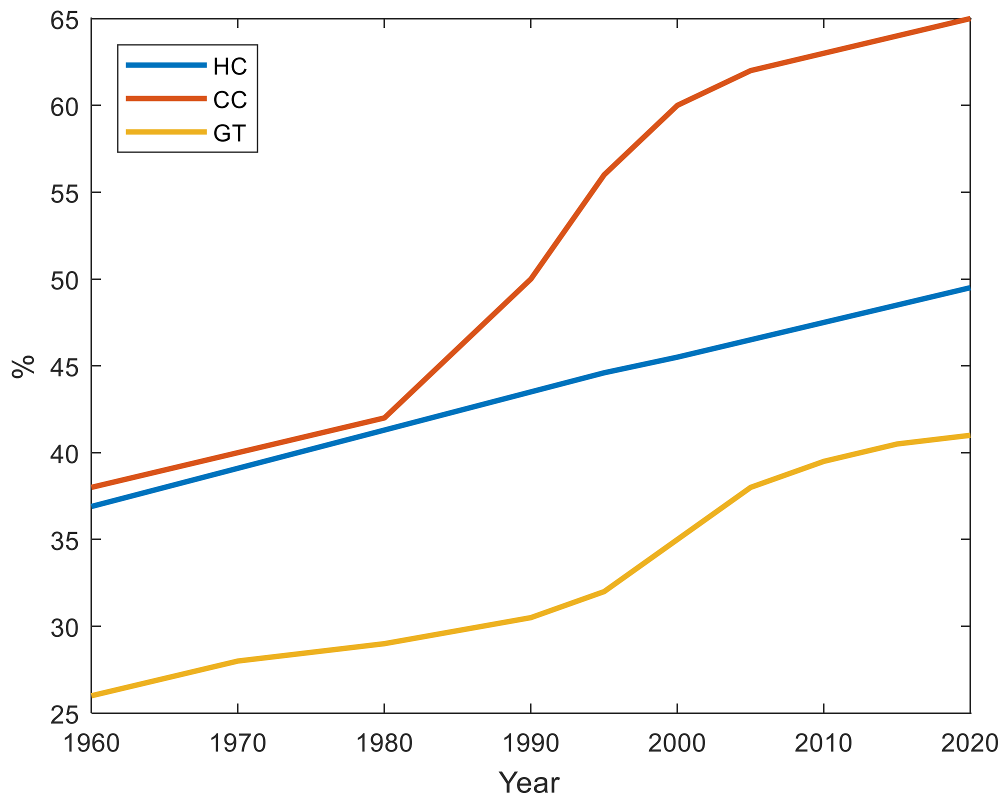

Fig. 2: Estimation of evolution of thermal efficiencies for the technologies in the test generation system.

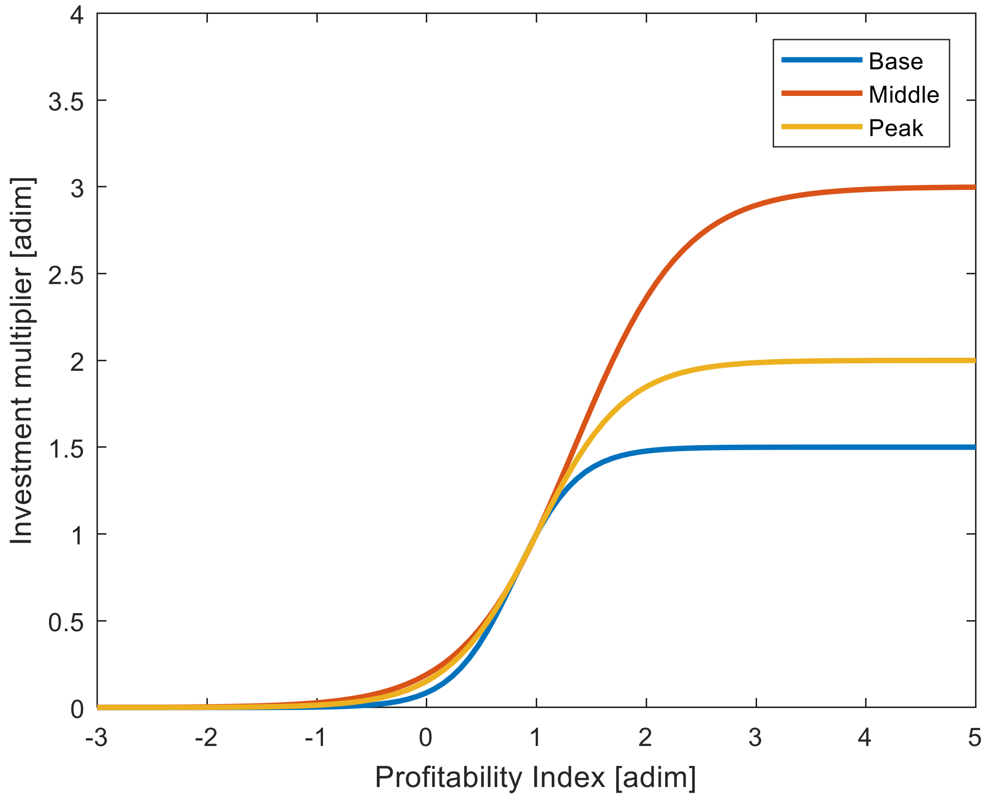

Fig. 3: Logistic curves representing the investment multiplier as a function of the profitability index for each technology.

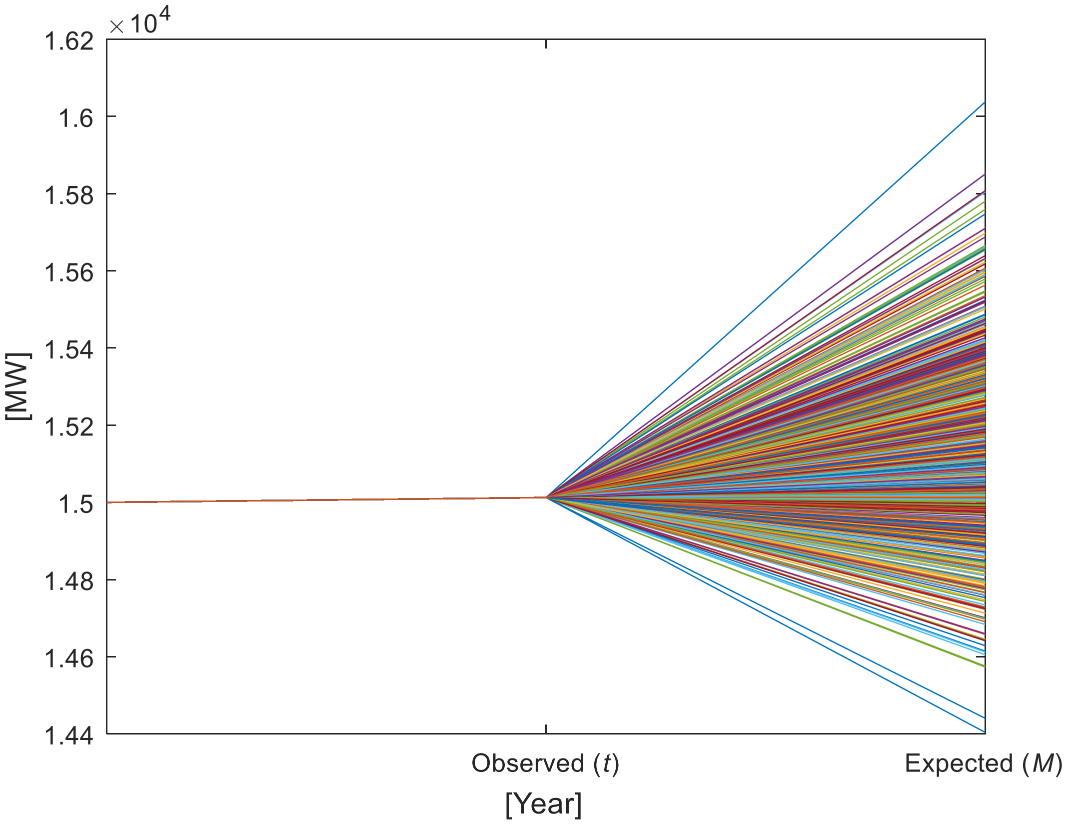

Fig. 4: Simulation of the observed and expected maximum demand at time t after the beginning of simulations.

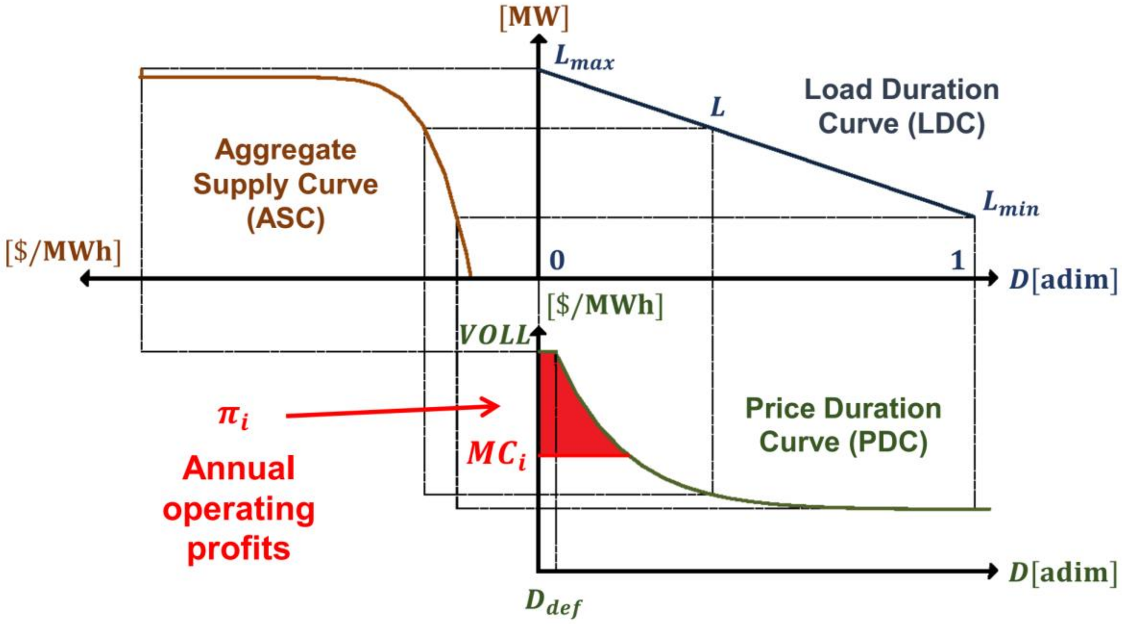

Fig. 5: Exemplary definition of a Price Duration Curve (PDC).

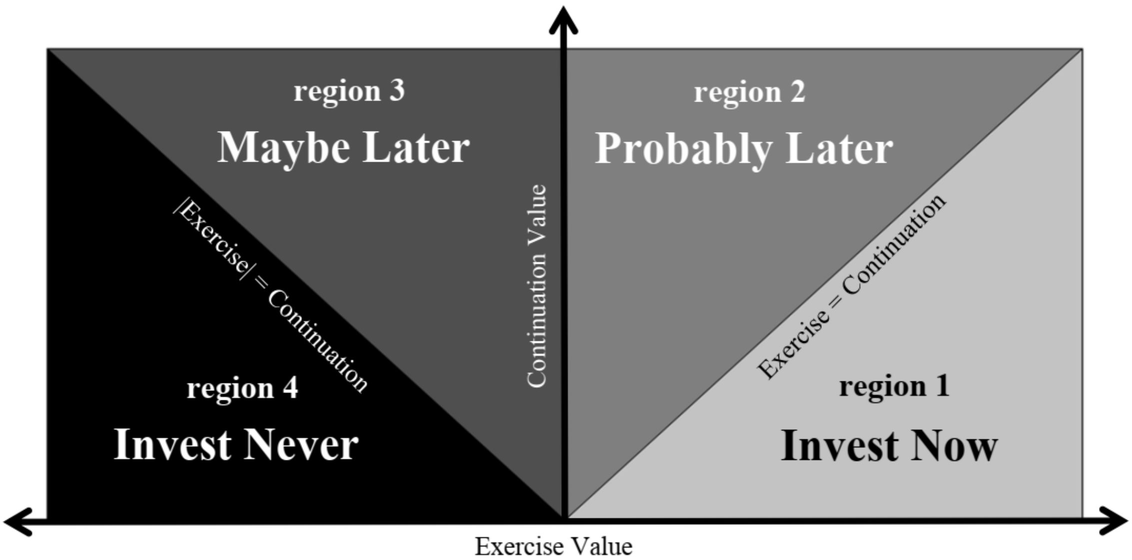

Fig. 6: Investment decision regions in the Exercise-Continuation Value plane.

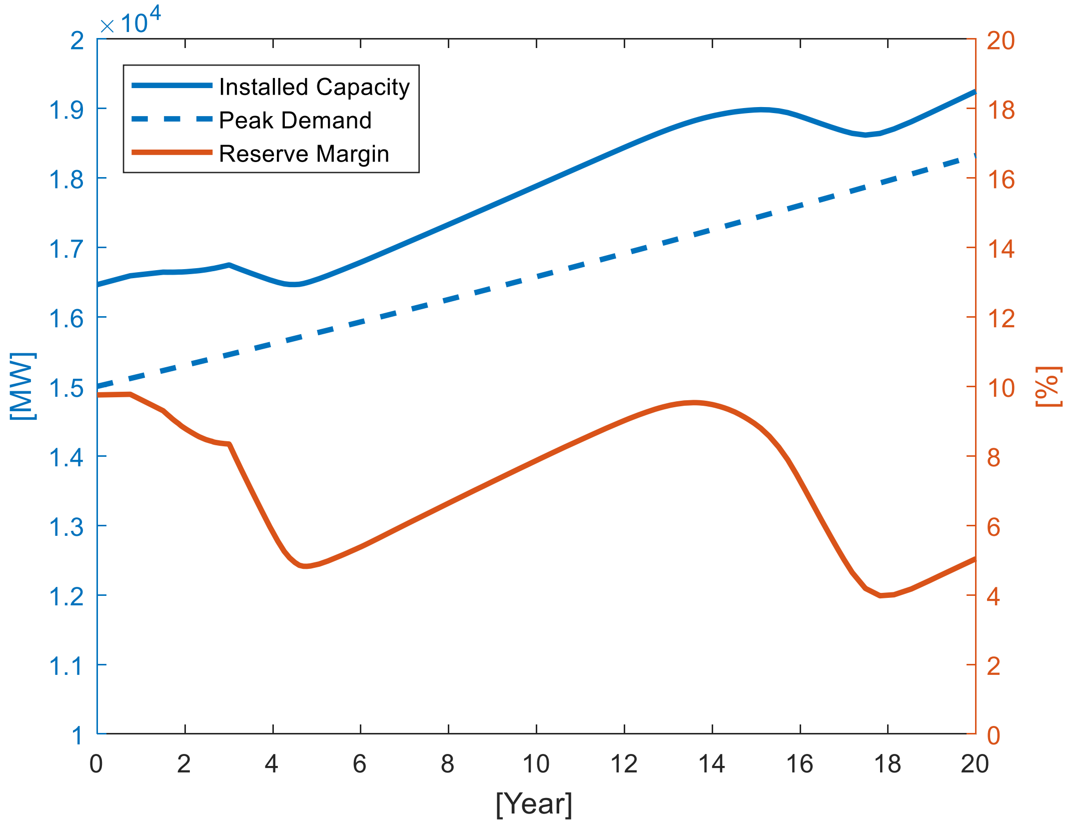

Fig. 7: Simulation of installed capacity and reserve margins.

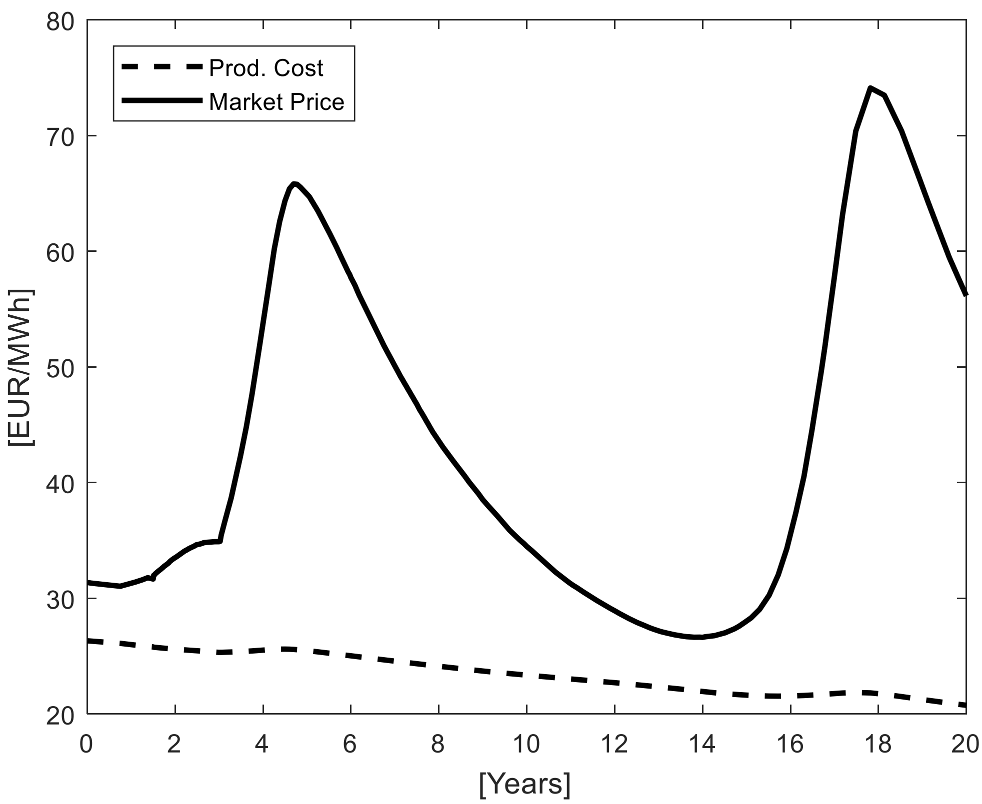

Fig. 8: Simulation of the expected annual production cost and market price.

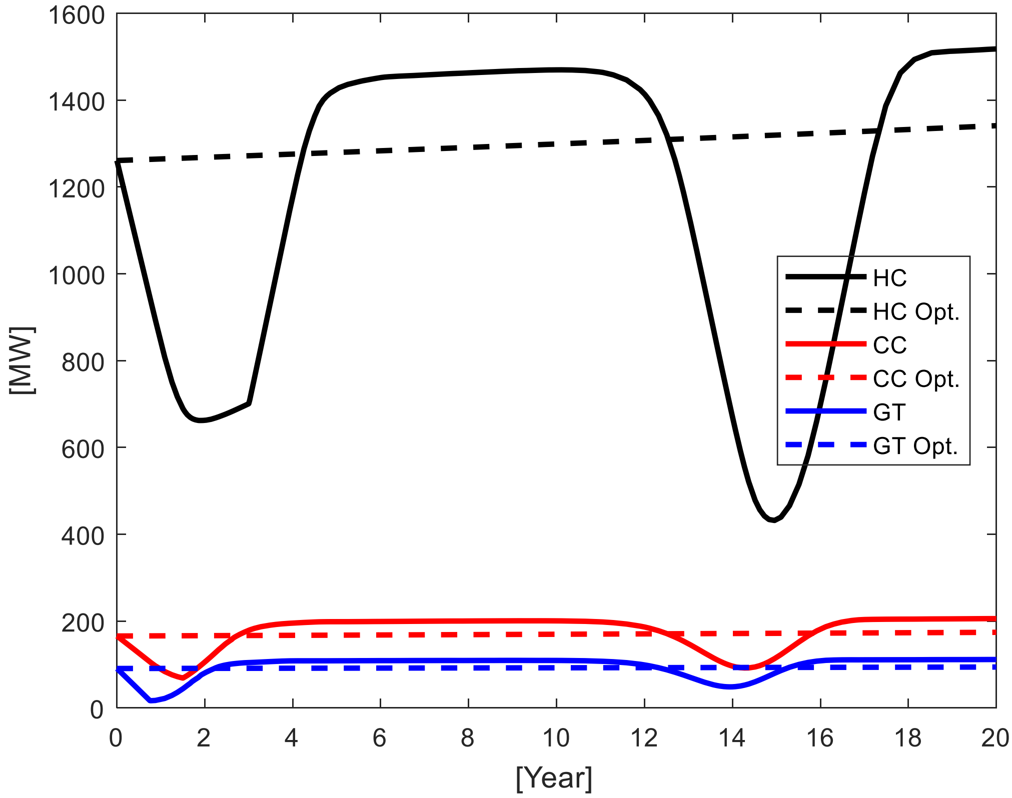

Fig. 9: Simulation of the capacity under construction for the three considered technologies.

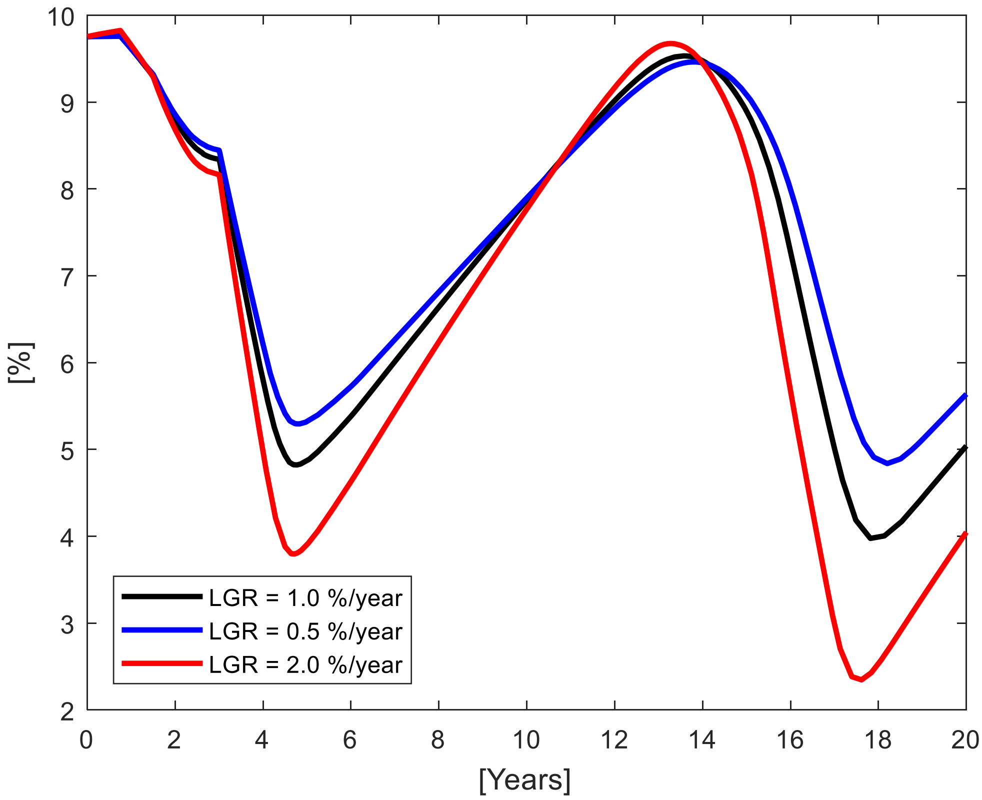

Fig. 10: Simulation of reserve margins with different long-term demand growth rates.

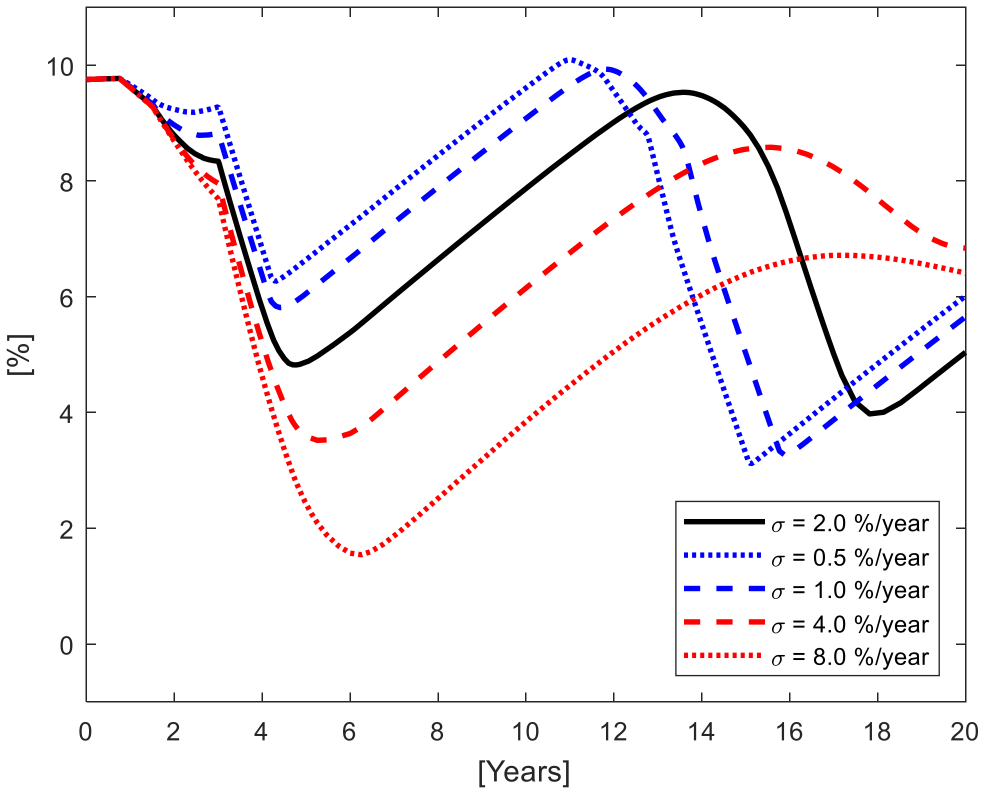

Fig. 11: Simulation of reserve margins with different volatilities for the demand growth rate expected at the *Option Maturity*.

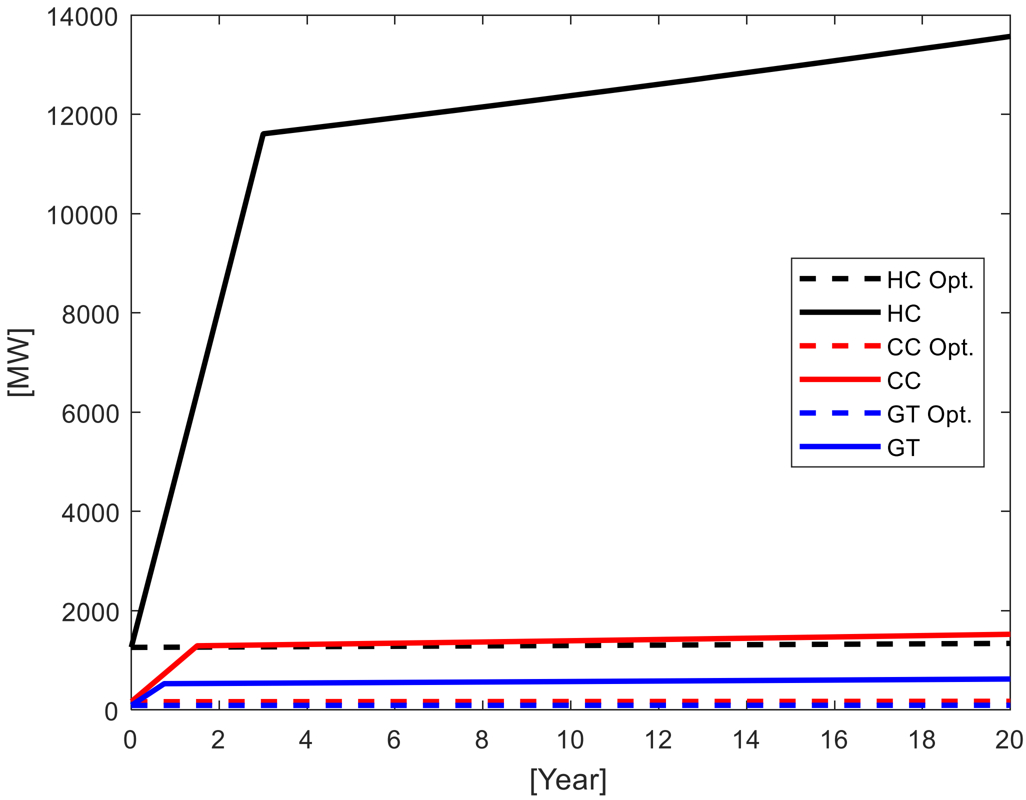

Fig. 12: Simulation of the capacity under construction for the three considered technologies with the first indirect extreme-condition test.

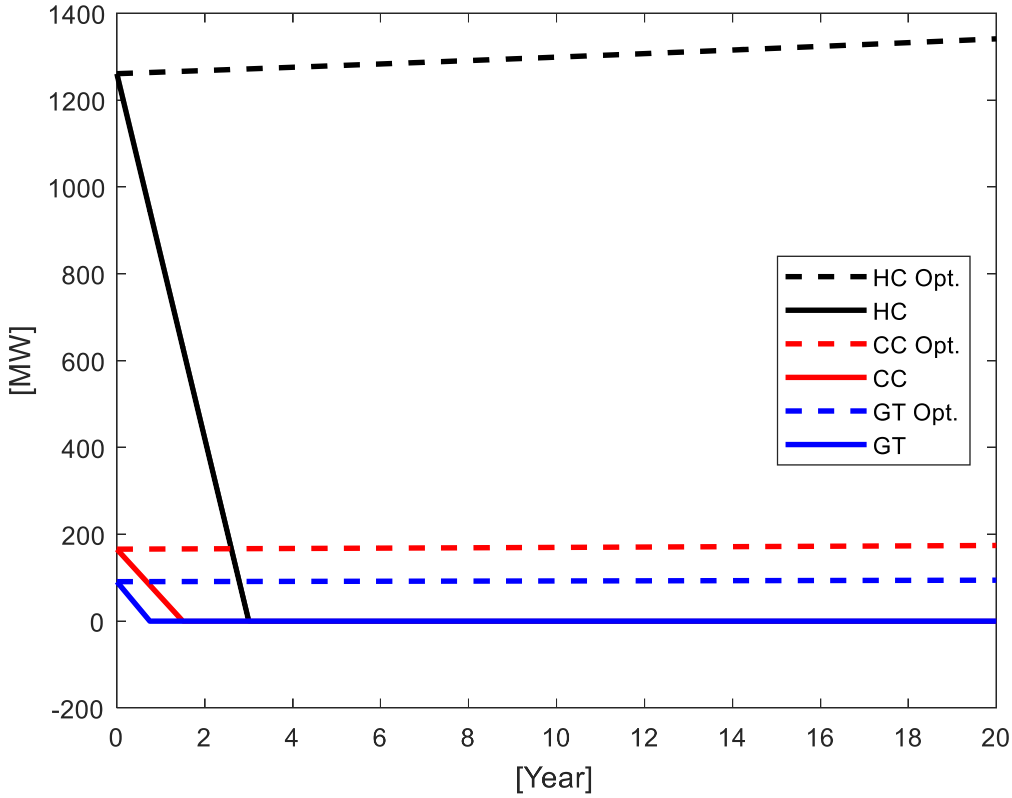

Fig. 13: Simulation of the capacity under construction for the three considered technologies with the second indirect extreme-condition test.

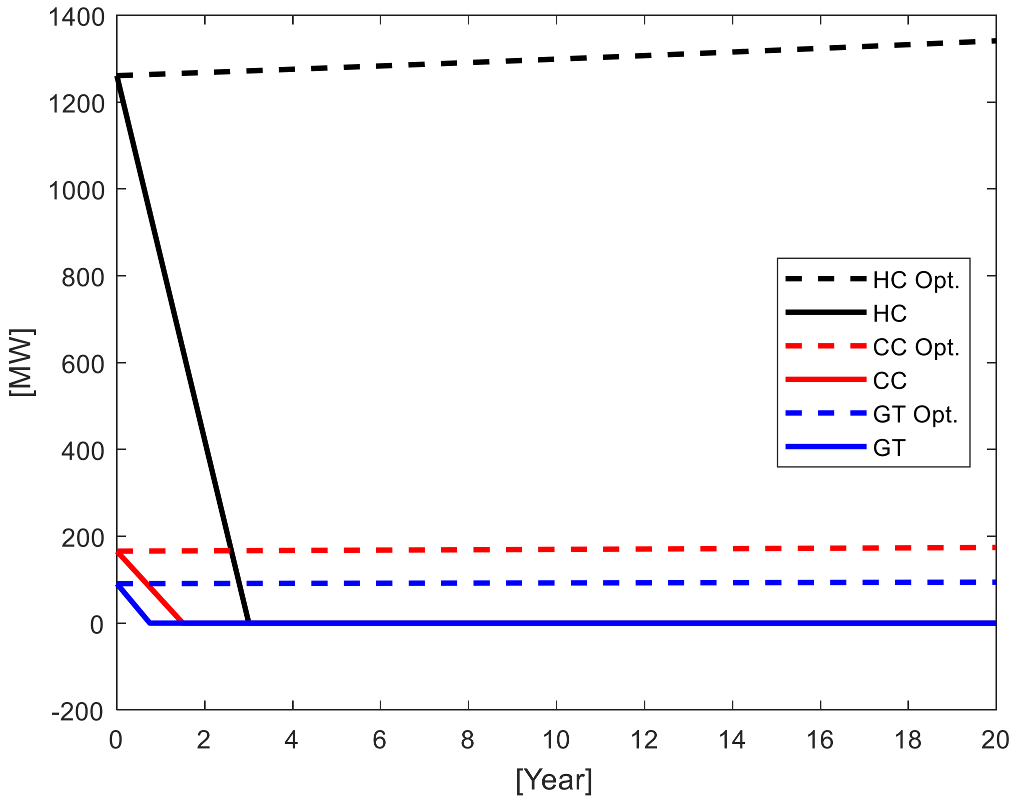

Fig. 14: Simulation of the capacity under construction for the three considered technologies with the third indirect extreme-condition test

Tables

TABLE I: Input data for the generation test system.

Technology	HC	CC	GT
Capacity (K_i) [MW]	11905	2540	2020
Construction delay ($\bar{T}_i^C$)[month]	36	18	9
Lifetime (T_i) [year]	40	30	20
Investment costs (IC_i) [€/kW]	1000	600	300
Amortization period ($\bar{T}_i^A$) [year]	25	20	15
Discount rate (ρ) [%/year]	12.5	12.5	12.5
Fixed costs [€/MWh]	14.92	9.33	5.06
Fuel price (FP_i) [€/MWh]	6.50	10.50	10.50
Efficiency (with age structure at $t = 0$) [adim]	0.4063	0.4625	0.3072
VOLL [€/MWh]	1000	1000	1000
Usage duration (D_i) (at $t = 0$) [adim]	0.8345	0.3723	0.0052
Unit availability (q) (at $t = 0$) [adim]	0.9096	0.9096	0.9096

TABLE II: Parameters of the logistic functions of investment responsiveness.

Parameters	HC	CC	GT
Saturation (im_i^{max})	1.5	3.0	2.0
Alpha (α_i)	3.5	2.0	2.5
Beta (β_i)	-2.8069	-2.6932	-2.5000

TABLE III: Parameters to describe the stochastic evolution of peak and minimum demand (g_L), total installed capacity (g_K), and fuel prices at the *Option Maturity*.

Parameters	g_L	g_K	g_{FP}^{coal}	g_{FP}^{gas}
Long-term growth rate ($\bar{g}$) [%/year]	1.00	1.00	0.02	0.02
Speed of reversion (η) [%/year]	50.0	50.0	50.0	50.0
Volatility (σ) [%/year]	2.00	2.00	1.85	3.95
Correlation [adim]	0.80	0.80	0.70	0.70

Table IV: Key parameters and results of the performed direct extreme-condition tests.

	Base case	Test #1: Large demand	Test #2: No demand	Test #2: Large costs
Test input variables				
$L_{max}(0)$ [MW]	15,000	225,000	600	15,000
$L_{min}(0)$ [MW]	10,000	150,000	400	10,000
ρ [%/year]	12.5	12.5	12.5	312.5
t [month]	2.25	2.25	2.25	2.25
Model output (ultimate)				
$K_T^{UC}(t)$ [MW]	1,350	2,350	1,344	1,344
Result	Base case	Increase	Decrease	Decrease
Pass/Fail	Base case	Pass	Pass	Pass

Appendix

Polynomial estimation of thermal efficiencies in Fig. 2 ($t > t_0$ and $t_0 = 2000$) (t in months)

HC: $+1.5684\text{e} - 24t^4 - 6.2935\text{e} - 22t^3 + 6.2014\text{e} - 20t^2 + 1.6667\text{e} - 04t + 0.455$

CC: $-3.2150\text{e} - 11t^4 + 1.9290\text{e} - 08t^3 - 4.0509\text{e} - 06t^2 + 5.1389\text{e} - 04t + 0.600$

GT: $-3.2150\text{e} - 11t^4 + 1.9290\text{e} - 08t^3 - 4.7454\text{e} - 06t^2 + 7.2222\text{e} - 04t + 0.350$